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A TREATISE

ON

ARCHITECTURE AND BUILDING CONSTRUCTION

PREPARED FOR STUDENTS OF
THE INTERNATIONAL CORRESPONDENCE SCHOOLS
SCRANTON, PA.

Volume VIII

ANSWERS TO QUESTIONS

First Edition

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A KEY

TO ALL THE

QUESTIONS AND EXAMPLES

CONTAINED IN THE

QUESTION PAPERS

INCLUDED IN VOLS. I–V.

The various Keys composing this volume have been given the same section numbers as the Question Papers to which they refer; and the answers and solutions have been numbered to correspond with the questions contained in the Question Papers. In many instances, the answer to a question would involve a repetition of statements given in the Instruction Papers; hence, in all such cases the student has been referred to an article in the Instruction Paper, the reading of which will enable him to answer the question himself.

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ARITHMETIC.

(QUESTIONS 1–75. SEC. 1.)

(1) See Art. **1.**

(2) See Art. **3.**

(3) See Arts. **5** and **6.**

(4) See Arts. **10** and **11.**

(5) 980 = Nine hundred eighty.
605 = Six hundred five.
28,284 = Twenty-eight thousand two hundred eighty-four.
9,006,042 = Nine million six thousand forty-two.
850,317,002 = Eight hundred fifty million three hundred seventeen thousand two.
700,004 = Seven hundred thousand four.

(6) Seven thousand six hundred = 7,600.
Eighty-one thousand four hundred two = 81,402.
Five million four thousand seven = 5,004,007.
One hundred eight million ten thousand one = 108,010,001.
Eighteen million six = 18,000,006.
Thirty thousand ten = 30,010.

(7) In adding whole numbers, place the numbers to be added directly under each other so that the extreme right-hand figures will stand in the same column, regardless of the position of those at the left. Add the first column of figures at the extreme right, which equals 19 units, or 1 ten and 9 units. We place

3 2 9 0	
5 0 4	
8 6 5 4 0 3	
2 0 7 4	
8 1	
7	
8 7 1 3 5 9	Ans.

9 units under the units column, and reserve 1 ten for the column of tens. $1+8+7+9 = 25$ tens, or 2 hundreds and 5 tens. Place 5 tens under the tens column, and reserve 2 hundreds for the hundreds column. $2+4+5+2 = 13$ hundreds, or 1 thousand and 3 hundreds. Place 3 hundreds under the hundreds column, and reserve the 1 thousand for the thousands column. $1+2+5+3 = 11$ thousands, or 1 ten thousand and 1 thousand. Place the 1 thousand in the column of thousands, and reserve the 1 ten thousand for the column of ten thousands. $1+6 = 7$ ten thousands. Place this 7 ten thousands in the ten thousands column. There is but one figure, 8, in the hundreds of thousands place in the numbers to be added, so it is placed in the hundreds of thousands column of the sum.

A simple (though less scientific) explanation of the same problem is the following: $7+1+4+3+4+0 = 19$; write the 9 and reserve the 1. $1+8+7+0+0+9 = 25$; write the 5 and reserve the 2. $2+0+4+5+2 = 13$; write the 3 and reserve the 1. $1+2+5+3 = 11$; write the 1 and reserve 1. $1+6 = 7$; write the 7. Bring down the 8 to its place in the sum.

(8)

$$\begin{array}{r} 709 \\ 8304725 \\ 391 \\ 100302 \\ 300 \\ 909 \\ \hline 8407336 \end{array}$$ Ans.

(9) (*a*) In subtracting whole numbers, place the subtrahend, or smaller number, under the minuend, or larger number, so that the right-hand figures stand directly under each other. Begin at the *right* to subtract. We cannot subtract 8 units from 2 units, so we take 1 ten from the 6 tens and add it to the 2 units. 1 ten = 10 units, so we have 10 units + 2 units = 12 units. Then 8 units from 12 units leaves 4 units. We took 1 ten from 6 tens, so only 5

tens remain. 3 tens from 5 tens leaves 2 tens. In the hundreds column we have 3 hundreds from 9 hundreds leaves 6 hundreds. We cannot subtract 3 thousands from 0 thousands, so we take 1 ten thousand from 5 ten thousands and add it to the 0 thousands. 1 ten thousand = 10 thousands, and 10 thousands + 0 thousands = 10 thousands. Subtracting, we have 3 thousands from 10 thousands leaves 7 thousands. We took 1 ten thousand from 5 ten thousands and have 4 ten thousands remaining. Since there are no ten thousands in the subtrahend, the 4 in the ten thousands column in the minuend is brought down into the same column in the remainder, because 0 from 4 leaves 4.

$$\begin{array}{r} 50962 \\ 3338 \\ \hline 47624 \end{array}$$ Ans.

(*b*)
$$\begin{array}{r} 15339 \\ 10001 \\ \hline 5338 \end{array}$$ Ans.

(10) (*a*)
$$\begin{array}{r} 70968 \\ 32975 \\ \hline 37993 \end{array}$$ Ans.

(*b*)
$$\begin{array}{r} 100000 \\ 98735 \\ \hline 1265 \end{array}$$ Ans.

(11) We have given the minuend or greater number (1,004) and the difference or remainder (49). Placing these in the usual form of subtraction, we have $\begin{array}{r} 1004 \\ \text{——} \\ \hline 49 \end{array}$ in which the dash (——) represents the number sought. This number is evidently *less* than 1,004 by the difference 49, hence, 1,004 − 49 = 955, the smaller number. For the sum of the two numbers we then have

$$\begin{array}{rl} 1004 & \textit{larger} \\ 955 & \textit{smaller} \\ \hline 1959 & \textit{sum.} \end{array}$$ Ans.

Or, this problem may be solved as follows: If the greater of two numbers is 1,004, and the difference between them is 49, then it is evident that the smaller number must be equal to the difference between the greater number (1,004) and the difference (49); or, 1,004 − 49 = 955, the smaller

number. Since the greater number equals 1,004 and the smaller number equals 955, their sum equals 1,004 + 955 = 1,959. Ans.

(12) The numbers connected by the plus (+) sign must first be added. Performing these operations we have

5 9 6 2	3 8 7 4
8 4 7 1	2 0 3 9
9 0 2 3	5 9 1 3 *sum.*
2 3 4 5 6 *sum.*	

Subtracting the smaller number (5,913) from the greater (23,456), we have

2 3 4 5 6
5 9 1 3
1 7 5 4 3 *difference.* Ans.

(13) $4 4 6 7 5 = amount willed to his son.
2 6 3 8 0 = amount willed to his daughter.
$7 1 0 5 5 = amount willed to his two children.
$1 2 5 0 0 0 = amount willed to his wife and two children.
7 1 0 5 5 = amount willed to his two children.
$5 3 9 4 5 = amount willed to his wife. Ans.

(14) In the multiplication of whole numbers, place the multiplier under the multiplicand, and multiply each term of the multiplicand by each term of the multiplier, writing the right-hand figure of each product obtained under the term of the multiplier which produces it.

(*a*) 5 2 6 3 8 7
7
3 6 8 4 7 0 9 Ans.

7 times 7 units = 49 units, or 4 tens and 9 units. We write the 9 units and reserve the 4 tens. 7 times 8 tens = 56 tens; 56 + 4 tens reserved = 60 tens, or 6 hundreds and 0 tens. Write the 0 tens and reserve the 6 hundreds. 7 × 3 hundreds = 21 hundreds; 21 + 6 hundreds reserved = 27 hundreds, or 2 thousands and 7 hundreds. Write the 7 hundreds and reserve the 2 thousands. 7 × 6 thousands = 42 thousands;

42 + 2 thousands reserved = 44 thousands, or 4 ten thousands and 4 thousands. Write the 4 thousands and reserve the 4 ten thousands. 7 × 2 ten thousands = 14 ten thousands; 14 + 4 ten thousands reserved = 18 ten thousands, or 1 hundred thousand and 8 ten thousands. Write the 8 ten thousands and reserve the 1 hundred thousand. 7 × 5 hundred thousands = 35 hundred thousands; 35 + 1 hundred thousand reserved = 36 hundred thousands. Since there are no more figures in the multiplicand to be multiplied, we write the 36 hundred thousands in the product. This completes the multiplication.

A simpler (though less scientific) explanation of the same problem is the following:

7 times 7 = 49; write the 9 and reserve the 4. 7 times 8 = 56; 56 + 4 reserved = 60; write the 0 and reserve the 6. 7 times 3 = 21; 21 + 6 reserved = 27; write the 7 and reserve the 2. 7 × 6 = 42; 42 + 2 reserved = 44; write the 4 and reserve 4. 7 × 2 = 14; 14 + 4 reserved = 18; write the 8 and reserve the 1. 7 × 5 = 35; 35 + 1 reserved = 36; write the 36.

In this case the multiplier is 17 *units*, or 1 ten and 7 units, so that the product is obtained by adding two partial products, namely, 7 × 700,298 and 10 × 700,298. The actual operation is performed as follows:

(*b*)

	700298
	17
	4902086
	700298
	11905066 Ans.

7 times 8 = 56; write the 6 and reserve the 5. 7 times 9 = 63; 63 + 5 reserved = 68; write the 8 and reserve the 6. 7 times 2 = 14; 14 + 6 reserved = 20; write the 0 and reserve the 2. 7 times 0 = 0; 0 + 2 reserved = 2; write the 2. 7 times 0 = 0; 0 + 0 reserved = 0; write the 0. 7 times 7 = 49; 49 + 0 reserved = 49; write the 49.

To multiply by the 1 ten we say 1 times 700,298 = 700,298, and write 700,298 under the first partial product, as shown, with the right-hand figure 8 under the multiplier 1. Add the two partial products; their sum equals the entire product.

(*c*)
$$\begin{array}{r} 217 \\ 103 \\ \hline 651 \\ 2170 \\ \hline 22351 \\ 67 \\ \hline 156457 \\ 134106 \\ \hline 1497517 \end{array}$$

Multiply any two of the numbers together and multiply their product by the third number.

Ans.

(15) If your watch ticks every second, then to find how many times it ticks in 1 week, it is necessary to find the number of seconds in one week.

$$\begin{array}{r} 60 \\ 60 \\ \hline 3600 \\ 24 \\ \hline 14400 \\ 7200 \\ \hline 86400 \\ 7 \\ \hline 604800 \end{array}$$

60 seconds = 1 minute.
60 minutes = 1 hour.
3600 seconds = 1 hour.
24 hours = 1 day.
86400 seconds = 1 day.
7 days = 1 week.
604800 seconds in 1 week, or the number of times that your watch ticks in 1 week. Ans.

(16) If a monthly publication contains 24 pages, a yearly volume will contain 12×24, or 288 pages, since there are 12 months in one year; and eight yearly volumes will contain 8×288, or 2,304 pages.

$$\begin{array}{r} 24 \\ 12 \\ \hline 288 \\ 8 \\ \hline 2304 \end{array}$$

Ans.

(17) If an engine and boiler are worth \$3,246, and the building is worth 3 times as much, plus \$1,200, then the building is worth

$$\begin{array}{rr} & \$3246 \\ \times & 3 \\ \hline & 9738 \\ + & 1200 \\ \hline & \$10938 \end{array}$$

= value of building.

If the tools are worth twice as much as the building, plus $1,875, then the tools are worth

$$\begin{array}{rl} & \$10938 \\ \times & \underline{\qquad\quad 2} \\ & 21876 \\ + & \underline{\quad 1875} \\ & \$23751 = \text{value of tools.} \end{array}$$

$$\begin{array}{rl} \text{Value of building} = & \$10938 \\ \text{Value of tools} = & \underline{\quad 23751} \\ & \$34689 = \text{value of the building and tools. } (a) \text{ Ans.} \end{array}$$

$$\begin{array}{rl} \text{Value of engine and boiler} = & \$\quad 3246 \\ \text{Value of building and tools} = & \underline{\quad 34689} \\ & \$37935 = \text{value of the whole plant. } (b) \text{ Ans.} \end{array}$$

(18) (a) $(72\times48\times28\times5)\div(96\times15\times7\times6)$.

Placing the numerator over the denominator the problem becomes

$$\frac{72\times48\times28\times5}{96\times15\times7\times6} = ?$$

The 5 in the dividend and 15 in the divisor are both divisible by 5, since 5 divided by 5 equals 1, and 15 divided by 5 equals 3. Cross off the 5 and write the 1 over it; also, cross off the 15 and write the 3 under it. Thus,

$$\frac{72\times48\times28\times\overset{1}{\cancel{5}}}{96\times\underset{3}{\cancel{15}}\times7\times6} =$$

The 5 and 15 are not to be considered any longer, and, in fact, may be erased entirely and the 1 and 3 placed in their stead, and treated as if the 5 and the 15 never existed. Thus,

$$\frac{72\times48\times28\times1}{96\times3\times7\times6} =$$

72 in the dividend and 96 in the divisor are divisible by 12, since 72 divided by 12 equals 6, and 96 divided by 12 equals 8. Cross off the 72 and write the 6 over it; also, cross off the 96 and write the 8 under it. Thus,

$$\frac{\overset{6}{\cancel{72}}\times 48\times 28\times 1}{\underset{8}{\cancel{96}}\times 3\times 7\times 6} =$$

The 72 and 96 are not to be considered any longer, and, in fact, may be erased entirely and the 6 and 8 placed in their stead, and treated as if the 72 and 96 never existed. Thus,

$$\frac{6\times 48\times 28\times 1}{8\times 3\times 7\times 6} =$$

Again, 28 in the dividend and 7 in the divisor are divisible by 7, since 28 divided by 7 equals 4, and 7 divided by 7 equals 1. Cross off the 28 and write the 4 over it; also, cross off the 7 and write the 1 under it. Thus,

$$\frac{6\times 48\times \overset{4}{\cancel{28}}\times 1}{8\times 3\times \underset{1}{\cancel{7}}\times 6} =$$

The 28 and 7 are not to be considered any longer, and, in fact, may be erased entirely and the 4 and 1 placed in their stead, and treated as if the 28 and 7 never existed. Thus,

$$\frac{6\times 48\times 4\times 1}{8\times 3\times 1\times 6} =$$

Again, 48 in the dividend and 6 in the divisor are divisible by 6, since 48 divided by 6 equals 8, and 6 divided by 6 equals 1. Cross off the 48 and write the 8 over it; also, cross off the 6 and write the 1 under it. Thus,

$$\frac{6\times \overset{8}{\cancel{48}}\times 4\times 1}{8\times 3\times 1\times \underset{1}{\cancel{6}}} =$$

The 48 and 6 are not to be considered any longer, and, in fact, may be erased entirely and the 8 and 1 placed in their stead, and treated as if the 48 and 6 never existed. Thus,

$$\frac{6\times8\times4\times1}{8\times3\times1\times1} =$$

Again, 6 in the dividend and 3 in the divisor are divisible by 3, since 6 divided by 3 equals 2, and 3 divided by 3 equals 1. Cross off the 6 and write the 2 over it; also, cross off the 3 and write the 1 under it. Thus,

$$\frac{\overset{2}{\cancel{6}}\times8\times4\times1}{8\times\underset{1}{\cancel{3}}\times1\times1} =$$

The 6 and 3 are not to be considered any longer, and, in fact, may be erased entirely and the 2 and 1 placed in their stead, and treated as if the 6 and 3 never existed. Thus,

$$\frac{2\times8\times4\times1}{8\times1\times1\times1} =$$

Canceling the 8 in the dividend and the 8 in the divisor, the result is

$$\frac{2\times\overset{1}{\cancel{8}}\times4\times1}{\underset{1}{\cancel{8}}\times1\times1\times1} = \frac{2\times1\times4\times1}{1\times1\times1\times1}.$$

Since there are no two remaining numbers (one in the dividend and one in the divisor) divisible by any number except 1, without a remainder, it is impossible to cancel further.

Multiply all the uncanceled numbers in the dividend together, and divide their product by the product of all the uncanceled numbers in the divisor. The result will be the quotient. The product of all the uncanceled numbers in the dividend equals $2\times1\times4\times1 = 8$; the

product of all the uncanceled numbers in the divisor equals $1\times1\times1\times1 = 1$.

Hence, $$\frac{2\times1\times4\times1}{1\times1\times1\times1} = \frac{8}{1} = 8. \quad \text{Ans.}$$

Or, $$\frac{\overset{2}{\cancel{72}}\times\overset{1}{\cancel{48}}\times\overset{4}{\cancel{28}}\times\overset{1}{\cancel{5}}}{\underset{8}{\cancel{96}}\times\underset{3}{\cancel{15}}\times\underset{1}{\cancel{7}}\times\underset{1}{\cancel{6}}} = \frac{8}{1} = 8. \quad \text{Ans.}$$

(*b*) $(80\times60\times50\times16\times14)\div(70\times50\times24\times20)$.

Placing the numerator over the denominator, the problem becomes

$$\frac{80\times60\times50\times16\times14}{70\times50\times24\times20} = ?$$

The 50 in the dividend and 70 in the divisor are both divisible by 10, since 50 divided by 10 equals 5, and 70 divided by 10 equals 7. Cross off the 50 and write the 5 over it; also, cross off the 70 and write the 7 under it. Thus,

$$\frac{80\times60\times\overset{5}{\cancel{50}}\times16\times14}{\underset{7}{\cancel{70}}\times50\times24\times20} =$$

The 50 and 70 are not to be considered any longer, and, in fact, may be erased entirely and the 5 and 7 placed in their stead, and treated as if the 50 and 70 never existed. Thus,

$$\frac{80\times60\times5\times16\times14}{7\times50\times24\times20} =$$

Also, 80 in the dividend and 20 in the divisor are divisible by 20, since 80 divided by 20 equals 4, and 20 divided by 20 equals 1. Cross off the 80 and write the 4 over it; also, cross off the 20 and write the 1 under it. Thus,

$$\frac{\overset{4}{\cancel{80}}\times60\times5\times16\times14}{7\times50\times24\times\underset{1}{\cancel{20}}} =$$

The 80 and 20 are not to be considered any longer, and, in fact, may be erased entirely and the 4 and 1 placed in their stead, and treated as if the 80 and 20 never existed. Thus,

$$\frac{4\times60\times5\times16\times14}{7\times50\times24\times1}=$$

Again, 16 in the dividend and 24 in the divisor are divisible by 8, since 16 divided by 8 equals 2, and 24 divided by 8 equals 3. Cross off the 16 and write the 2 over it; also, cross off the 24 and write the 3 under it. Thus,

$$\frac{4\times60\times5\times\overset{2}{\cancel{16}}\times14}{7\times50\times\underset{3}{\cancel{24}}\times1}=$$

The 16 and 24 are not to be considered any longer, and, in fact, may be erased entirely and the 2 and 3 placed in their stead, and treated as if the 16 and 24 never existed. Thus,

$$\frac{4\times60\times5\times2\times14}{7\times50\times3\times1}=$$

Again, 60 in the dividend and 50 in the divisor are divisible by 10, since 60 divided by 10 equals 6, and 50 divided by 10 equals 5. Cross off the 60 and write the 6 over it; also, cross off the 50 and write the 5 under it. Thus,

$$\frac{4\times\overset{6}{\cancel{60}}\times5\times2\times14}{7\times\underset{5}{\cancel{50}}\times3\times1}=$$

The 60 and 50 are not to be considered any longer, and, in fact, may be erased entirely and the 6 and 5 placed in their stead, and treated as if the 60 and 50 never existed. Thus,

$$\frac{4\times6\times5\times2\times14}{7\times5\times3\times1}=$$

The 14 in the dividend and 7 in the divisor are divisible by 7, since 14 divided by 7 equals 2, and 7 divided by 7 equals 1.

Cross off the 14 and write the 2 over it; also, cross off the 7 and write the 1 under it. Thus,

$$\frac{4\times6\times5\times2\times\overset{2}{\cancel{14}}}{\underset{1}{\cancel{7}}\times5\times3\times1} =$$

The 14 and 7 are not to be considered any longer, and, in fact, may be erased entirely and the 2 and 1 placed in their stead, and treated as if the 14 and 7 never existed. Thus,

$$\frac{4\times6\times5\times2\times2}{1\times5\times3\times1} =$$

The 5 in the dividend and the 5 in the divisor are divisible by 5, since 5 divided by 5 equals 1. Cross off the 5 of the dividend and write the 1 over it; also, cross off the 5 of the divisor and write the 1 under it. Thus,

$$\frac{4\times6\times\overset{1}{\cancel{5}}\times2\times2}{1\times\underset{1}{\cancel{5}}\times3\times1} =$$

The 5 in the dividend and 5 in the divisor are not to be considered any longer, and, in fact, may be erased entirely and 1 and 1 placed in their stead, and treated as if the 5 and 5 never existed. Thus,

$$\frac{4\times6\times1\times2\times2}{1\times1\times3\times1} =$$

The 6 in the dividend and 3 in the divisor are divisible by 3, since 6 divided by 3 equals 2, and 3 divided by 3 equals 1. Cross off the 6 and place the 2 over it; also, cross off the 3 and place the 1 under it. Thus,

$$\frac{4\times\overset{2}{\cancel{6}}\times1\times2\times2}{1\times1\times\underset{1}{\cancel{3}}\times1} =$$

The 6 and 3 are not to be considered any longer, and, in fact, may be erased entirely and 2 and 1 placed in

their stead, and treated as if the 6 and 3 never existed. Thus,

$$\frac{4\times2\times1\times2\times2}{1\times1\times1\times1} = \frac{32}{1} = 32. \quad \text{Ans.}$$

Hence,

$$\frac{\overset{4}{\cancel{8}}\overset{2}{\cancel{0}}\times\overset{6}{\cancel{6}}\overset{1}{\cancel{0}}\times\overset{5}{\cancel{5}}\overset{2}{\cancel{0}}\times\overset{2}{\cancel{1}}\overset{1}{\cancel{6}}\times\overset{2}{\cancel{1}}\overset{1}{\cancel{4}}}{\underset{1}{\cancel{7}}\overset{1}{\cancel{0}}\times\underset{1}{\cancel{5}}\overset{1}{\cancel{0}}\times\underset{1}{\cancel{2}}\overset{1}{\cancel{4}}\times\underset{1}{\cancel{2}}\overset{1}{\cancel{0}}} = \frac{4\times2\times1\times2\times2}{1\times1\times1\times1} = \frac{32}{1} = 32. \quad \text{Ans.}$$

(19) 28 acres of land at \$133 an acre would cost

$$\begin{array}{r} 28\times\$133 = \$3{,}724 \\ 28 \\ \hline 1064 \\ 266 \\ \hline \$3724 \end{array}$$

If a mechanic earns \$1,500 a year and his expenses are \$968 per year, then he would save \$1500 − \$968, or \$532 per year.

$$\begin{array}{r} 968 \\ \hline \$532 \end{array}$$

If he saves \$532 in 1 year, to save \$3,724 it would take as many years as \$532 is contained times in \$3,724, or 7 years.

$$\begin{array}{r} 532)3724(7 \text{ years.} \quad \text{Ans.} \\ 3724 \\ \hline \end{array}$$

(20) If the freight train ran 365 miles in one week, and 3 times as far lacking 246 miles the next week, then it ran (3×365 miles) − 246 miles, or 849 miles the second week. Thus,

$$\begin{array}{rl} & 365 \\ & 3 \\ \hline & 1095 \\ & 246 \\ \hline \textit{difference} & 849 \text{ miles.} \quad \text{Ans.} \end{array}$$

(21) The distance from Philadelphia to Pittsburg is 354 miles. Since there are 5,280 feet in 1 mile, in 354 miles there are $354 \times 5{,}280$ feet, or 1,869,120 feet. If the driving wheel of the locomotive is 16 feet in circumference, then in going from Philadelphia to Pittsburg, a distance of 1,869,120 feet, it will make $1{,}869{,}120 \div 16$, or 116,820 revolutions.

16) 1 8 6 9 1 2 0 (1 1 6 8 2 0 rev. Ans.
1 6
2 6
1 6
1 0 9
9 6
1 3 1
1 2 8
3 2
3 2

(22) (*a*) 5 7 6) 5 8 9 8 2 4 (1 0 2 4 Ans.
5 7 6
1 3 8 2
1 1 5 2
2 3 0 4
2 3 0 4

(*b*) 4 3 9 1 1) 3 6 9 7 3 0 6 2 0 (8 4 2 0 Ans.
3 5 1 2 8 8
1 8 4 4 2 6
1 7 5 6 4 4
8 7 8 2 2
8 7 8 2 2

(*c*) 5 0 5) 2 5 2 7 5 2 5 (5 0 0 5 Ans.
2 5 2 5
2 5 2 5
2 5 2 5

(*d*)
$$
\begin{array}{r}
1234)4961794302(4020903 \text{ Ans.}\\
4936\\
\hline
2579\\
2468\\
\hline
11143\\
11106\\
\hline
3702\\
3702
\end{array}
$$

(23) The harness evidently cost the difference between \$444 and the amount which he paid for the horse and wagon. Since \$264 + \$153 = \$417, the amount paid for the horse and wagon, \$444 − \$417 = \$27, the cost of the harness.

$$
\begin{array}{r}
\$264\\
153\\
\hline
\$417
\end{array}
\qquad\qquad
\begin{array}{r}
\$444\\
417\\
\hline
\$27 \text{ Ans.}
\end{array}
$$

(24)
$$
\begin{array}{rrr}
(a) & (b) & (c)\\
1024 & 5005 & 43911\\
576 & 505 & 8420\\
\hline
6144 & 25025 & 878220\\
7168 & 2502250 & 175644\\
5120 & 25275225 \text{ Ans.} & 351288\\
\hline
5898824 \text{ Ans.} & & 369730620 \text{ Ans.}
\end{array}
$$

(25) Since there are 12 months in a year, the number of days the man works is $25 \times 12 = 300$ days. As he works 10 hours each day, the number of hours that he works in one year is $300 \times 10 = 3{,}000$ hours. Hence, he receives for his work $3{,}000 \times 30 = 90{,}000$ cents, or $90{,}000 \div 100 = \$900$. Ans.

(26) See Art. **71.**

(27) See Art. **77.**

(28) See Art. **73.**

(29) See Art. **73.**

(30) See Art. **75.**

(31) $\frac{13}{8}$ is an improper fraction, since its numerator, 13, is greater than its denominator, 8.

(32) $4\frac{1}{2}$; $14\frac{3}{10}$; $85\frac{4}{19}$.

(33) To reduce a fraction to its lowest terms means to change its form without changing its value. In order to do this, we must divide both numerator and denominator by the same number until we can no longer find any number (except 1) which will divide both of these terms without a remainder.

To reduce the fraction $\frac{4}{8}$ to its lowest terms, we divide both numerator and denominator by 4, and obtain as a result the fraction $\frac{1}{2}$. Thus, $\frac{4 \div 4}{8 \div 4} = \frac{1}{2}$; similarly, $\frac{4 \div 4}{16 \div 4} = \frac{1}{4}$; $\frac{8 \div 4}{32 \div 4} = \frac{2 \div 2}{8 \div 2} = \frac{1}{4}$; $\frac{32 \div 8}{64 \div 8} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2}$. Ans.

(34) When the denominator of any number is not expressed, it is understood to be 1, so that $\frac{6}{1}$ is the same as $6 \div 1$, or 6. To reduce $\frac{6}{1}$ to an improper fraction whose denominator is 4, we must multiply both numerator and denominator by some number which will make the denominator of 6 equal to 4. Since this denominator is 1, by multiplying both terms of $\frac{6}{1}$ by 4 we shall have $\frac{6 \times 4}{1 \times 4} = \frac{24}{4}$, which has the *same value* as 6, but has a *different form.* Ans.

(35) In order to reduce a mixed number to an improper fraction, we must multiply the whole number by the denominator of the fraction and add the numerator of the fraction to that product. This *result* is the numerator of the improper fraction, of which the denominator is the denominator of the fractional part of the mixed number.

$7\frac{7}{8}$ means the same as $7 + \frac{7}{8}$. In 1 there are $\frac{8}{8}$, hence in 7 there are $7 \times \frac{8}{8} = \frac{56}{8}$; $\frac{56}{8}$ plus the $\frac{7}{8}$ of the mixed number $= \frac{56}{8} + \frac{7}{8} = \frac{63}{8}$, which is the required improper fraction.

$$13\tfrac{5}{16} = \frac{(13 \times 16) + 5}{16} = \tfrac{213}{16}; \quad 10\tfrac{3}{4} = \frac{(10 \times 4) + 3}{4} = \tfrac{43}{4}.$$

(36) The value of a fraction is obtained by dividing the numerator by the denominator.

To obtain the value of the fraction $\frac{13}{2}$ we divide the numerator, 13, by the denominator, 2. 2 is contained in 13, 6 times, with 1 remaining. This 1 remaining is written

over the denominator, 2, thereby making the fraction $\frac{1}{2}$, which is annexed to the whole number, 6, and we obtain $6\frac{1}{2}$ as the mixed number. The reason for performing this operation is the following: In 1 there are $\frac{2}{2}$ (two halves), and in $\frac{13}{2}$ (thirteen halves) there are as many ones (1) as 2 is contained times in 13, which is 6, and $\frac{1}{2}$ (one-half) remaining. Hence, $\frac{13}{2} = 6 + \frac{1}{2} = 6\frac{1}{2}$, the required mixed number.

$\frac{17}{4} = 4\frac{1}{4}$; $\frac{69}{16} = 4\frac{5}{16}$; $\frac{16}{8} = 2$; $\frac{67}{64} = 1\frac{3}{64}$. Ans.

(37) In division of fractions, invert the divisor (or, in other words, turn it upside down) and then proceed as in multiplication.

(*a*) $35 \div \frac{5}{16} = \frac{35}{1} \times \frac{16}{5} = \frac{35 \times 16}{1 \times 5} = \frac{560}{5} = 112$. Ans.

(*b*) $\frac{9}{16} \div 3 = \frac{9}{16} \div \frac{3}{1} = \frac{9}{16} \times \frac{1}{3} = \frac{9 \times 1}{16 \times 3} = \frac{9}{48} = \frac{3}{16}$. Ans.

(*c*) $\frac{17}{2} \div 9 = \frac{17}{2} \div \frac{9}{1} = \frac{17}{2} \times \frac{1}{9} = \frac{17 \times 1}{2 \times 9} = \frac{17}{18}$. Ans.

(*d*) $\frac{113}{64} \div \frac{7}{16} = \frac{113}{64} \times \frac{16}{7} = \frac{113 \times 16}{64 \times 7} = \frac{1808}{448} = \frac{452}{112} = \frac{113}{28}$.

$28)\overline{113}\,(4\frac{1}{28}$. Ans.
$\underline{112}$
1

(*e*) $15\frac{3}{4} \div 4\frac{3}{8} = ?$ Before proceeding with the division, reduce both of the mixed numbers to improper fractions. Thus, $15\frac{3}{4} = \frac{(15 \times 4) + 3}{4} = \frac{60 + 3}{4} = \frac{63}{4}$, and $4\frac{3}{8} = \frac{(4 \times 8) + 3}{8} = \frac{32 + 3}{8} = \frac{35}{8}$. The problem is now $\frac{63}{4} \div \frac{35}{8} = ?$ As before, invert the divisor and multiply; $\frac{63}{4} \div \frac{35}{8} = \frac{63}{4} \times \frac{8}{35} = \frac{63 \times 8}{4 \times 35} = \frac{504}{140} = \frac{252}{70} = \frac{126}{35} = \frac{18}{5}$.

$5)\overline{18}\,(3\frac{3}{5}$. Ans.
$\underline{15}$
3

(38) $\frac{1}{8} + \frac{2}{8} + \frac{5}{8} = \frac{1 + 2 + 5}{8} = \frac{8}{8} = 1$. Ans.

When the denominators of the fractions to be added are alike, we know that the units are divided into the same

number of parts (in this case eighths); we, therefore, add the numerators of the fractions to find the number of parts (eighths) taken or considered, thereby obtaining $\frac{8}{8}$ or 1 as the sum.

(39) When the denominators are *not* alike we know that the units are divided into unequal parts, so before adding them we must find a common denominator for the denominators of all the fractions. Reduce the fractions to fractions having this common denominator, add the numerators, and write the sum over the common denominator.

In this case, the least common denominator, or the least number that will contain all the denominators, is 16; hence, we must reduce all these fractions to 16ths and then add their numerators.

$\frac{1}{4}+\frac{3}{8}+\frac{5}{16}=?$ To reduce the fraction $\frac{1}{4}$ to a fraction having 16 for a denominator, we must multiply both terms of the fraction by some number which will make the denominator 16. This number evidently is 4; hence, $\frac{1\times4}{4\times4}=\frac{4}{16}$.

Similarly, both terms of the fraction $\frac{3}{8}$ must be multiplied by 2 to make the denominator 16, and we have $\frac{3\times2}{8\times2}=\frac{6}{16}$.

The fractions now have a common denominator, 16; hence, we find their sum by adding the numerators and placing their sum over the common denominator, thus: $\frac{4}{16}+\frac{6}{16}+\frac{5}{16}=\frac{4+6+5}{16}=\frac{15}{16}$. Ans.

(40) When mixed numbers and whole numbers are to be added, add the fractional parts of the mixed numbers separately, and if the resulting fraction is an improper fraction, reduce it to a whole or mixed number. Next, add all the whole numbers, including the one obtained from the addition of the fractional parts, and annex to their sum the fraction of the mixed number obtained from reducing the improper fraction.

$42+31\frac{5}{8}+9\frac{7}{16}=?$ Reducing $\frac{5}{8}$ to a fraction having a denominator of 16, we have $\frac{5\times2}{8\times2}=\frac{10}{16}$. Adding the two

fractional parts of the mixed numbers, we have $\frac{10}{16}+\frac{7}{16}$ $=\frac{10+7}{16}=\frac{17}{16}=1\frac{1}{16}$.

The problem now becomes $42+31+9+1\frac{1}{16}=?$

42
31
9
$1\frac{1}{16}$

$83\frac{1}{16}$ Ans.

Adding all the whole numbers and the number obtained from adding the fractional parts of the mixed numbers, we obtain $83\frac{1}{16}$ as their sum.

(41) $29\frac{3}{4}+50\frac{5}{8}+41+69\frac{3}{16}=?$ $\frac{3}{4}=\frac{3\times4}{4\times4}=\frac{12}{16}$.

$\frac{5}{8}=\frac{5\times2}{8\times2}=\frac{10}{16}$. $\frac{12}{16}+\frac{10}{16}+\frac{3}{16}=\frac{12+10+3}{16}=\frac{25}{16}=1\frac{9}{16}$.

The problem now becomes $29+50+41+69+1\frac{9}{16}=?$

29 square inches.
50 square inches.
41 square inches.
69 square inches.
$1\frac{9}{16}$ square inches.

$190\frac{9}{16}$ square inches. Ans.

(42) (*a*) $\frac{7}{\frac{3}{16}}=7\div\frac{3}{16}=7\times\frac{16}{3}=\frac{7\times16}{3}=\frac{112}{3}=37\frac{1}{3}$. Ans.

The line between 7 and $\frac{3}{16}$ means that the 7 is to be divided by $\frac{3}{16}$.

(*b*) $\frac{\frac{15}{32}}{\frac{5}{8}}=\frac{15}{32}\div\frac{5}{8}=\frac{15}{32}\times\frac{8}{5}=\frac{\overset{3}{\cancel{15}}\times\overset{1}{\cancel{8}}}{\underset{4}{\cancel{32}}\times\cancel{5}}=\frac{3}{4}$. Ans.

(*c*) $\frac{\frac{4+3}{2+6}}{5}=\frac{\frac{7}{8}}{5}=\frac{7}{8\times5}=\frac{7}{40}$. (See Art. **131.**) Ans.

(43) $\frac{7}{8}$ = value of the fraction, and 28 = the numerator. We find that 4 multiplied by 7 = 28, so multiplying 8, the denominator of the fraction, by 4, we have 32 for the required denominator, and $\frac{28}{32}=\frac{7}{8}$. Hence, 32 is the required denominator. Ans.

(44) (*a*) $\frac{7}{8}-\frac{7}{16}=?$ When the denominators of fractions are not alike, it is evident that the units are divided into unequal parts; therefore, before subtracting, reduce the fractions to fractions having a common denominator. Then, subtract the numerators and place the remainder over the common denominator.

$$\frac{7\times2}{8\times2}=\tfrac{14}{16}.\quad \tfrac{14}{16}-\tfrac{7}{16}=\frac{14-7}{16}=\tfrac{7}{16}.\quad \text{Ans.}$$

(*b*) $13-7\frac{7}{16}=?$ This problem may be solved in two ways:

First: $13=12\frac{16}{16}$, since $\frac{16}{16}=1$, and $12\frac{16}{16}=12+\frac{16}{16}=12+1=13$.

$$\begin{array}{r} 1\ 2\frac{16}{16} \\ 7\frac{7}{16} \\ \hline 5\frac{9}{16} \end{array}$$

We can now subtract the whole numbers separately, and the fractions separately, and obtain $12-7=5$, and $\frac{16}{16}-\frac{7}{16}=\frac{16-7}{16}=\frac{9}{16}$. $5+\frac{9}{16}=5\frac{9}{16}$. Ans.

Second: By reducing both numbers to improper fractions having a denominator of 16.

$13=\frac{13}{1}=\frac{13\times16}{1\times16}=\frac{208}{16}$. $7\frac{7}{16}=\frac{(7\times16)+7}{16}=\frac{112+7}{16}=\frac{119}{16}$.

Subtracting, we have $\frac{208}{16}-\frac{119}{16}=\frac{208-119}{16}=\frac{89}{16}$ and $\frac{89}{16}$, the same result that was obtained by the first method. Ans.

$$\begin{array}{l} =16)\,89\,(\,5\frac{9}{16} \\ \quad\ \underline{80} \\ \quad\ \ 9 \end{array}$$

(*c*) $312\frac{9}{16}-229\frac{5}{32}=?$ We first reduce the fractions of the two mixed numbers to fractions having a common denominator. Doing this, we have $\frac{9}{16}=\frac{9\times2}{16\times2}=\frac{18}{32}$. We can now subtract the whole numbers and fractions separately, and have $312-229=83$, and $\frac{18}{32}-\frac{5}{32}=\frac{18-5}{32}=\frac{13}{32}$.

$$\begin{array}{r} 3\ 1\ 2\frac{18}{32} \\ 2\ 2\ 9\frac{5}{32} \\ \hline 8\ 3\frac{13}{32} \end{array}$$

$83+\frac{13}{32}=83\frac{13}{32}$. Ans.

(45) The man evidently traveled $85\frac{5}{12}+78\frac{9}{15}+125\frac{17}{35}$ miles. Adding the fractions separately in this case,

$$\frac{5}{12}+\frac{9}{15}+\frac{17}{35}=\frac{5}{12}+\frac{3}{5}+\frac{17}{35}=\frac{175+252+204}{420}=\frac{631}{420}=1\frac{211}{420}.$$

Adding the whole numbers and the mixed number representing the sum of the fractions, the sum is $289\frac{211}{420}$ miles. Ans.

To find the least common denominator, we have

$$\begin{array}{r} 5\,)\ 12,\quad 5,\quad 35 \\ \hline 7\,)\ 12,\quad 1,\quad 7 \\ \hline 12,\quad 1,\quad 1, \text{ or } 5\times 7\times 12 = 420. \end{array}$$

$$\begin{array}{r} 85 \\ 78 \\ 125 \\ 1\frac{211}{420} \\ \hline 289\frac{211}{420} \end{array}$$

(46) $5\ 7\ 3\frac{4}{5}$ tons. $\quad \frac{4}{5}=\frac{32}{40}$

$2\ 1\ 6\frac{5}{8}$ tons. $\quad \frac{5}{8}=\frac{25}{40}$

difference $3\ 5\ 7\frac{7}{40}$ tons. Ans. $\quad \frac{7}{40}=$ *difference.*

(47) Reducing $9\frac{1}{4}$ to an improper fraction, it becomes $\frac{37}{4}$. Multiplying $\frac{37}{4}$ by $\frac{3}{8}$, $\frac{37}{4}\times\frac{3}{8}=\frac{111}{32}=3\frac{15}{32}$ dollars. Ans.

(48) Referring to Arts. **114** and **116**,

$\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{7}{11}$ of $\frac{19}{20}$ of 11 multiplied by $\frac{7}{8}$ of $\frac{5}{6}$ of 45

$$=\frac{\overset{1}{\cancel{2}}\times\overset{1}{\cancel{3}}\times 7\times 19\times\cancel{11}\times 7\times 5\times\overset{3}{\cancel{45}}}{\underset{1}{\cancel{3}}\times 4\times\cancel{11}\times\underset{4}{\cancel{20}}\times 1\times 8\times\underset{1}{\cancel{6}}\times\underset{1}{\cancel{1}}}=\frac{7\times 19\times 7\times 5\times 3}{4\times 4\times 8}=\frac{13{,}965}{128}$$

$=109\frac{13}{128}$. Ans.

(49) $\frac{3}{4}$ of $16=\frac{3}{\cancel{4}}\times\frac{\overset{4}{\cancel{16}}}{1}=12.$ $\quad 12\div\frac{2}{3}=\frac{\overset{6}{\cancel{12}}}{1}\times\frac{3}{\cancel{2}}=18.$ Ans.

(50) $211\frac{1}{4}\times 1\frac{7}{8}=\frac{845}{4}\times\frac{15}{8}$, reducing the mixed numbers to improper fractions. $\frac{845}{4}\times\frac{15}{8}=\frac{12{,}675}{32}$ cents = amount paid for the lead. The number of pounds sold is evidently

$$\frac{12{,}675}{32}\div 2\tfrac{1}{2}=\frac{\overset{2{,}535}{\cancel{12{,}675}}}{\underset{16}{\cancel{32}}}\times\frac{2}{\cancel{5}}=\frac{2{,}535}{16}=158\tfrac{7}{16}\text{ pounds.}$$ The

amount remaining is $211\frac{1}{4} - 158\frac{7}{16} = \frac{845}{4} - \frac{2,535}{16} = \frac{3,380}{16} - \frac{2,535}{16} = \frac{845}{16} = 52\frac{13}{16}$ pounds. Ans.

(51)

	tenths.	hundredths.	thousandths.	ten-thousandths.	hundred-thousandths.	millionths.	
.0	8						= *Eight* **hundredths.**
.1	3	1					= *One hundred thirty-one* **thousandths.**
.0	0	0	1				= *One* **ten-thousandth.**
.0	0	0	0	2	7		= *Twenty-seven* **millionths.**
.0	1	0	8				= *One hundred eight* **ten-thousandths.**

	tenths.	hundredths.	thousandths.	ten-thousandths.	
93.0	1	0	1		= Ninety-three, and *one hundred one* **ten-thousandths.**

In reading decimals, read the number just as you would if there were no ciphers before it. Then count from the decimal point towards the right, beginning with tenths, to as many places as there are figures, and the *name* of the last figure must be annexed to the previous reading of the figures to give the decimal reading. Thus, in the first example above, the simple reading of the figure is *eight*, and the name of its position in the decimal scale is **hundredths**, so that the decimal reading is *eight* **hundredths.** Similarly, the figures in the fourth example are ordinarily read *twenty-seven;* the name of the position of the figure 7 in the decimal scale is **millionths,** giving, therefore, the decimal reading as *twenty-seven* **millionths.**

If there should be a whole number before the decimal point, read it as you would read any whole number, and read the decimal as you would if the whole number were not there; or, read the whole number and then say, "and" so many hundredths, thousandths, or whatever it may be, as "ninety-three, and one hundred one ten-thousandths."

(52) See Art. **139.**

(53) See Art. **153.**

(54) See Art. **160.**

(55) A fraction is one or more of the equal parts of a unit, and is expressed by a numerator and a denominator, while a decimal fraction is a number of tenths, hundredths, thousandths, etc. of a unit, and is expressed by placing a period (.), called a decimal point, to the left of the figures of the number, and omitting the denominator.

(56) See Art. **165.**

(57) To reduce the fraction $\frac{1}{2}$ to a decimal, we annex one cipher to the numerator, which makes it 1.0. Dividing 1.0, the numerator, by 2, the denominator, gives a quotient of .5, the decimal point being placed before the *one* figure of the

quotient, or .5, since only one cipher was annexed to the numerator.

$\frac{7}{8}$) 7.0 0 0
.8 7 5

$\frac{5}{32}$) 5.0 0 0 0 0 (.1 5 6 2 5 Ans.
3 2
1 8 0
1 6 0
2 0 0
1 9 2
8 0
6 4
1 6 0
1 6 0

Since $.65 = \frac{65}{100}$, then, $\frac{65}{100}$ must equal .65. Or, when the denominator is 10, 100, 1,000, etc., point off as many places in the numerator as there are ciphers in the denominator. Doing so, $\frac{65}{100} = .65$. Ans.

$\frac{125}{1000} = .125$. Ans.

(58) (*a*) This example, written in the form of a fraction, means that the numerator (32.5 + .29 + 1.5) is to be divided by the denominator (4.7 + 9). The operation is as follows:

$$\frac{32.5 + .29 + 1.5}{4.7 + 9} = ?$$

3 2.5
\+ .2 9
\+ 1.5
1 3.7) 3 4.2 9 0 0 0 (2.5 0 2 9 Ans.
2 7 4
6 8 9
6 8 5
4 0 0
2 7 4
1 2 6 0
1 2 3 3
2 7

4.7
\+ 9.0
1 3.7

Since there are 5 decimal places in the dividend, and 1 in the divisor, there are $5-1$ or 4 places to be pointed off in the quotient. The fifth figure of the decimal is evidently less than 5.

(*b*) Here again the problem is to divide the numerator, which is $(1.283\times\overline{8+5})$, by the denominator, which is 2.63. The operation is as follows:

$$\frac{1.283\times\overline{8+5}}{2.63} = ? \quad \overline{8+5} = 13.$$

```
    1.283
  ×   13
   3849
   1283
-------------
2.63 ) 16.679000 ( 6.3418   Ans.
       1578
       ----
         899
         789
         ---
         1100
         1052
         ----
           480
           263
           ---
           2170
           2104
           ----
             66
```

(*c*) $$\frac{\overline{589+27}\times\overline{163-8}}{25+39} = ?$$

```
  589          163
+  27        -   8
  ---          ---
  616          155
             × 616
               ---
               930
              155
             930
             ----
            95480
```

$$
\begin{array}{r}
25 \\
+39 \\
\hline
64
\end{array}
$$

$$
\begin{array}{r}
64)9548.0000(1491.875 \\
64 \\
\hline
314 \\
256 \\
\hline
588 \\
576 \\
\hline
120 \\
64 \\
\hline
560 \\
512 \\
\hline
480 \\
448 \\
\hline
320 \\
320
\end{array}
$$

There are three decimal places in the quotient, since three ciphers were annexed to the dividend.

(*d*) $\dfrac{\overline{40.6+7.1}\times(3.029-1.874)}{6.27+8.53-8.01}=?$

$$
\begin{array}{r}
40.6 \\
+\ 7.1 \\
\hline
47.7
\end{array}
$$

$$
\begin{array}{r}
6.27 \\
+\ 8.53 \\
\hline
14.80 \\
-\ 8.01 \\
\hline
6.79
\end{array}
$$

$$
\begin{array}{r}
3.029 \\
-1.874 \\
\hline
1.155 \\
\times\ 47.7 \\
\hline
8085 \\
8085 \\
4620 \\
\hline
6.79)55.093500(8.1139\ \text{Ans.} \\
5432 \\
\hline
773 \\
679 \\
\hline
945 \\
679 \\
\hline
2660 \\
2037 \\
\hline
6230 \\
6111 \\
\hline
119
\end{array}
$$

6 decimal places in the dividend − 2 decimal places in the divisor = 4 decimal places to be pointed off in the quotient.

(59) $.875 = \frac{875}{1000} = \frac{175}{200} = \frac{7}{8}$ of a foot.

1 foot = 12 inches.

$$\frac{7}{8} \text{ of 1 foot} = \frac{7}{\overset{2}{8}} \times \frac{\overset{3}{12}}{1} = \frac{21}{2} = 10\frac{1}{2} \text{ inches.} \quad \text{Ans.}$$

(60) 12 inches = 1 foot.

$$\frac{3}{16} \text{ of an inch} = \frac{3}{16} \div 12 = \frac{\overset{1}{3}}{16} \times \frac{1}{\underset{4}{12}} = \frac{1}{64} \text{ of a foot.}$$

$\frac{1}{64}$) 1.0 0 0 0 0 0 (.0 1 5 6 2 5 Ans.

6 4

3 6 0

3 2 0

4 0 0

3 8 4

1 6 0

1 2 8

3 2 0

3 2 0

Point off 6 decimal places in the quotient, since we annexed six ciphers to the dividend, the divisor containing no decimal places; hence, 6 − 0 = 6 places to be pointed off.

(61) If 1 cubic inch of water weighs .03617 of a pound, the weight of 1,500 cubic inches will be .03617 × 1,500 = 54.255 lb.

.0 3 6 1 7 lb.

1 5 0 0

1 8 0 8 5 0 0

3 6 1 7

5 4.2 5 5 0 0 lb. Ans.

(62) 72.6 feet of fencing at $.50 a foot would cost

7 2.6 × .50, or $36.30.

.5 0

$ 3 6.3 0 0

If, by selling a carload of coal at a profit of $1.65 per ton, I make $36.30, then there must be as many tons

of coal in the car as 1.65 is contained times in 36.30, or 22 tons.

```
1.65 ) 36.30 ( 22 tons.  Ans.
       330
       ---
        330
        330
        ---
```

(63)
```
231 ) 17892.000000 ( 77.454554, or 77.4545
      1617                  to four decimal
       1722                 places. Ans.
       1617
       ----
        1050
         924
         ---
         1260
         1155
         ----
          1050
           924
           ---
           1260
           1155
           ----
            1050
```

(64)
$$\frac{37.13 \quad 2 \quad .0952}{\frac{74.26}{1} \times \frac{24}{1} \times \frac{3.1416}{1} \times 19 \times 19 \times 350} \over \frac{33,000}{1,000} \times \frac{12}{1} \times \frac{4}{2}$$

$$= \frac{37.13 \times .0952 \times 19 \times 19 \times 350}{1,000} = \frac{446,618\ 947600}{1,000}$$

$= 446.619$ to three decimal places. Ans.

```
  37.13      19        361          3.534776
  .0952      19        350           126350
  -----     ---       ----         ---------
   7426      171      18050       1767388800
 18565        19      1083         106043328
33417        361     126350        21208656
-------                            7069552
3.534776                          3534776
                                  ---------
                                  446618.947600
```

(65) See Art. **174.** Applying rule in Art. **175,**

(*a*) $.7928 \times \frac{64}{64} = \frac{50.7392}{64} = \frac{51}{64}$. Ans.

(*b*) $.1416 \times \frac{32}{32} = \frac{4.5312}{32} = \frac{5}{32}$. Ans.

(*c*) $.47915 \times \frac{16}{16} = \frac{7.6664}{16} = \frac{8}{16} = \frac{1}{2}$. Ans.

(66) In subtraction of decimals, place the decimal points directly under each other, and proceed as in the subtraction of whole numbers, placing the decimal point in the remainder directly under the decimal points above.

(*a*)
$$\begin{array}{r} 709.6300 \\ .8514 \\ \hline 708.7786 \end{array}$$ Ans.

In the above example we proceed as follows: We cannot subtract 4 ten-thousandths from 0 ten-thousandths, and as there are no thousandths, we take 1 hundredth from the 3 hundredths. 1 hundredth = 10 thousandths = 100 ten-thousandths. 4 ten-thousandths from 100 ten-thousandths leaves 96 ten-thousandths. 96 ten-thousandths = 9 thousandths + 6 ten-thousandths. Write the 6 ten-thousandths in the ten-thousandths place in the remainder. The next figure in the subtrahend is 1 thousandth. This must be subtracted from the 9 thousandths, which is a part of the 1 hundredth taken previously from the 3 hundredths. Subtracting, we have 1 thousandth from 9 thousandths leaves 8 thousandths, the 8 being written in its place in the remainder. Next we have to subtract 5 hundredths from 2 hundredths (1 hundredth having been taken from the 3 hundredths makes it but 2 hundredths now). Since we cannot do this, we take 1 tenth from 6 tenths. 1 tenth (= 10 hundredths) + 2 hundredths = 12 hundredths. 5 hundredths from 12 hundredths leaves 7 hundredths. Write the 7 in the hundredths place in the remainder. Next we have to subtract 8 tenths from 5 tenths (5 tenths now, because 1 tenth was taken from the 6 tenths). Since this cannot be done, we take 1 unit from the 9 units. 1 unit = 10 tenths; 10 tenths + 5 tenths = 15 tenths, and 8 tenths from 15 tenths leaves 7 tenths. Write the 7 in the

tenths place in the remainder. In the minuend we now have 708 units (1 unit having been taken away) and 0 units in the subtrahend. 0 units from 708 units leaves 708 units; hence, we write 708 in the remainder.

(*b*) 81.963 − 1.700 = 80.263 Ans.

(*c*) 18.00 − .18 = 17.82 Ans.

(*d*) 1.000 − .001 = .999 Ans.

(*e*) $872.1-(.8721+.008)=?$ In this problem we are to subtract $(.8721+.008)$ from 872.1. First perform the operation as indicated by the sign between the decimals enclosed by the parenthesis.

.8721
.0080
.8801 *sum.*

Subtracting the sum (obtained by adding the decimals enclosed within the parenthesis) from the number 872.1 (as required by the minus sign before the parenthesis), we obtain the required remainder.

872.1000
− .8801
871.2199 Ans.

(*f*) $(5.028+.0073)-(6.704-2.38)=?$ First perform the operations as indicated by the signs between the numbers enclosed by the parentheses. The first parenthesis shows that 5.028 and .0073 are to be added. This gives 5.0353 as their sum.

5.0280
.0073
5.0353 *sum.*

The second parenthesis shows that 2.38 is to be subtracted from 6.704. The difference is found to be 4.324.

6.704
− 2.380
4.324 *difference.*

The sign between the parentheses indicates that the quantities obtained by performing the above operations are to be subtracted, namely, that 4.324 is to be subtracted from 5.0353. Performing this operation, we obtain .7113 as the final result.

5.0353
− 4.3240
.7113 Ans.

(67) In subtracting a decimal from a fraction, or in subtracting a fraction from a decimal, either reduce the

fraction to a decimal before subtracting or reduce the decimal to a fraction and then subtract.

(*a*) $\frac{7}{8} - .807 = ?$ $\frac{7}{8}$ reduced to a decimal becomes $8\overline{)7.000}$ .875

.875
.807
.068 Ans.

Subtracting .807 from .875, the remainder is .068, as shown.

(*b*) $.875 - \frac{3}{8} = ?$ Reducing .875 to a fraction, we have $.875 = \frac{875}{1000} = \frac{175}{200} = \frac{35}{40} = \frac{7}{8}$; hence, $\frac{7}{8} - \frac{3}{8} = \frac{7-3}{8} = \frac{4}{8} = \frac{1}{2}$, or, .5. Ans.

Or, by reducing $\frac{3}{8}$ to a decimal, $8\overline{)3.000}$ .375 and then subtracting, we obtain $.875 - .375 = .5 = \frac{5}{10} = \frac{1}{2}$, the same answer as above.

.877
.375
.500 Ans.

(*c*) $(\frac{5}{32} + .435) - (\frac{21}{100} - .07) = ?$ We first perform the operations as indicated by the signs between the numbers enclosed by the parentheses. Reduce $\frac{5}{32}$ to a decimal and we obtain $\frac{5}{32} = .15625$ (see example 60).

Adding .15625 and .435,

.15625
.435
sum .59125

$\frac{21}{100} = .21$; subtracting,

.21
.07
difference .14

We are now prepared to perform the operation indicated by the minus sign between the parentheses, which is,

.59125
.14
.45125 Ans.

(*d*) This problem means that 33 millionths and 17 thousandths are to be added. Also, that 53 hundredths and 274 thousandths are to be added, and the smaller of these sums is to be subtracted from the larger sum. Thus $(.53 + .274) - (.000033 + .017) = ?$

tenths.	hundredths.	thousandths.	ten-thousandths.	hundred-thousandths.	millionths.		tenths.	hundredths.	thousandths.		
.0	0	0	0	3	3		.5	3			
.0	1	7					.2	7	4		
.0	1	7	0	3	3	*sum.*	.8	0	4	*sum.*	

.8 0 4 *larger sum.*
.0 1 7 0 3 3 *smaller sum.*
difference .7 8 6 9 6 7 Ans.

(68) In addition of decimals the decimal points must be placed directly under each other, so that tenths will come under tenths, hundredths under hundredths, thousandths under thousandths, etc. The addition is then performed as in whole numbers, the decimal point of the sum being placed directly under the decimal points above.

.1 2 5
.7
.0 8 9
.4 0 0 5
.9
.0 0 0 0 2 7
2.2 1 4 5 2 7 Ans.

(69)
9 2 7.4 1 6
8.2 7 4
3 7 2.6
6 2.0 7 9 3 8
1 3 7 0.3 6 9 3 8 Ans.

(70)

tenths.	hundredths.	thousandths.	ten-thousandths.	hundred-thousandths.	millionths.	
.0	1	7				
.2						
.0	0	0	0	4	7	
.2	1	7	0	4	7	Ans.

(71) (*a*) There are 3 decimal places in the multiplicand and 3 in the multiplier; hence, there are 3 + 3 or 6 decimal places in the product. Since the product contains but four figures, we prefix two ciphers in order to obtain the necessary six decimal places.

.1 0 7
.0 1 3
3 2 1
1 0 7
.0 0 1 3 9 1 Ans.

(*b*)

```
    2 0 3
    2.0 3
    ------
    6 0 9
  4 0 6 0
  -------
  4 1 2.0 9
    .2 0 3
  ---------
  1 2 3 6 2 7
8 2 4 1 8 0
-----------
8 3.6 5 4 2 7   Ans.
```

There are 2 decimal places in the multiplier and none in the multiplicand; hence, there are 2 + 0 or 2 decimal places in the first product.

Since there are 2 decimal places in the multiplicand and 3 decimal places in the multiplier, there are 3 + 2 or 5 decimal places in the second product.

(*c*) First perform the operations indicated by the signs between the numbers enclosed by the parenthesis, and then perform whatever may be required by the sign before the parenthesis.

Multiply together the numbers 2.7 and 31.85.

```
  3 1.8 5
      2.7
  -------
  2 2 2 9 5
  6 3 7 0
  -------
  8 5.9 9 5
```

The parenthesis shows that .316 is to be taken from 3.16.

```
  3.1 6 0
   .3 1 6
  -------
  2.8 4 4
```

The product obtained by the first operation is now multiplied by the remainder obtained by performing the operation indicated by the signs within the parenthesis.

```
    8 5.9 9 5
      2.8 4 4
  -------------
    3 4 3 9 8 0
  3 4 3 9 8 0
  6 8 7 9 6 0
1 7 1 9 9 0
-----------
2 4 4.5 6 9 7 8 0   Ans.
```

(*d*) $(107{:}8 + 6.541 - 31.96) \times 1.742 = ?$

```
  1 0 7.8
+   6.5 4 1
  ---------
  1 1 4.3 4 1
-   3 1.9 6
  ---------
    8 2.3 8 1
```

```
    8 2.3 8 1
  x   1.7 4 2
  -------------
    1 6 4 7 6 2
  3 2 9 5 2 4
5 7 6 6 6 7
  8 2 3 8 1
-------------
1 4 3.5 0 7 7 0 2   Ans.
```

(72) (*a*) $(\frac{7}{16}-.13)\times\overline{.625+\frac{5}{8}}=?$

First perform the operation indicated by the parenthesis.

$\frac{7}{16}=\frac{7}{16}$) 7.0 0 0 0 (.4 3 7 5

6 4

6 0

4 8

1 2 0

1 1 2

8 0

8 0

We point off four decimal places, since we annex four ciphers.

.4 3 7 5

.1 3

Subtracting, we obtain .3 0 7 5

The vinculum has the same meaning as the parenthesis; hence, we perform the operation indicated by it. We point off three decimal places, since three ciphers were annexed to the 5.

$\frac{5}{8}=\frac{5}{8}$) 5.0 0 0

.6 2 5

Adding the terms included by the vinculum, we obtain

.6 2 5

.6 2 5

1.2 5 0

The final operation is to perform the work indicated by the sign between the parenthesis and the vinculum, thus,

.3 0 7 5

1.2 5

1 5 3 7 5

6 1 5 0

3 0 7 5

.3 8 4 3 7 5 Ans.

(*b*) $(\frac{19}{32}\times.21)-(.02\times\frac{3}{16})=?$

$.21=\frac{21}{100}$. $\frac{19}{32}\times\frac{21}{100}=\frac{399}{3200}$. $.02=\frac{2}{100}$. $\frac{2}{100}\times\frac{3}{16}=\frac{6}{1600}$

$=\frac{3}{800}$. $\frac{3}{800}=\frac{3\times4}{800\times4}=\frac{12}{3200}$. $\frac{399}{3200}-\frac{12}{3200}$

$=\frac{399-12}{3,200}=\frac{387}{3200}$.

Reducing $\frac{387}{3200}$ to a decimal, we obtain

$$
\begin{array}{r}
387 \\
\hline
3{,}200\ )\ 3\,8\,7.0\,0\,0\,0\,0\,0\,0\,0\ (.1\,2\,0\,9\,3\,7\,5 \quad \text{Ans.} \\
3\,2\,0\,0 \\
\hline
6\,7\,0\,0 \\
6\,4\,0\,0 \\
\hline
3\,0\,0\,0\,0 \\
2\,8\,8\,0\,0 \\
\hline
1\,2\,0\,0\,0 \\
9\,6\,0\,0 \\
\hline
2\,4\,0\,0\,0 \\
2\,2\,4\,0\,0 \\
\hline
1\,6\,0\,0\,0 \\
1\,6\,0\,0\,0
\end{array}
$$

Point off seven decimal places, since seven ciphers were annexed to the dividend.

(*c*) $(\frac{13}{4}+.013-2.17)\times\overline{13\frac{1}{4}-7\frac{5}{16}}=?$

$$
\begin{array}{r}
\frac{13}{4}=\frac{13}{4}\ )\ 1\,3.0\,0 \\
\hline
3.2\,5
\end{array}
$$

Point off two decimal places, since two ciphers were annexed to the dividend.

$$
\begin{array}{r}
3.2\,5 \\
+\ .0\,1\,3 \\
\hline
3.2\,6\,3 \\
-2.1\,7 \\
\hline
1.0\,9\,3
\end{array}
$$

$\frac{5}{16}$ reduced to a decimal is .3125, since

$$
\begin{array}{r}
\frac{5}{16}\ )\ 5.0\,0\,0\,0\ (.3\,1\,2\,5 \\
4\,8 \\
\hline
2\,0 \\
1\,6 \\
\hline
4\,0 \\
3\,2 \\
\hline
8\,0 \\
8\,0
\end{array}
$$

Point off four decimal places, since four ciphers were annexed to the dividend.

Then, $7\frac{5}{16}=7.3125$, and $13\frac{1}{4}=13.25$, since $\frac{1}{4}=\frac{1}{4}\ )\ 1.0\,0$ .25

$$
\begin{array}{r}
1\,3.2\,5 \\
-\ 7.3\,1\,2\,5 \\
\hline
5.9\,3\,7\,5
\end{array}
$$

$$
\begin{array}{r}
5.9\,3\,7\,5 \\
\times\quad 1.0\,9\,3 \\
\hline
1\,7\,8\,1\,2\,5 \\
5\,3\,4\,3\,7\,5 \\
5\,9\,3\,7\,5\,0 \\
\hline
6.4\,8\,9\,6\,8\,7\,5 \quad \text{Ans.}
\end{array}
$$

(73) (*a*) $.875 \div \frac{1}{2} = .875 \div .5$ (since $\frac{1}{2} = .5$) $= 1.75$. Ans.

Another way of solving this is to reduce .875 to its equivalent common fraction and then divide.

$.875 = \frac{7}{8}$, since $.875 = \frac{875}{1000} = \frac{175}{200} = \frac{35}{40} = \frac{7}{8}$; then,

$$\frac{7}{8} \div \frac{1}{2} = \frac{7}{\not 8_4} \times \frac{\not 2}{1} = \frac{7}{4} = 1\tfrac{3}{4}.$$

Since $\frac{3}{4} = \frac{3}{4}$) 3.0 0 (.7 5, $1\frac{3}{4} = 1.75$, the same answer as above.

2 8

2 0

2 0

(*b*) $\frac{7}{8} \div .5 = \frac{7}{8} \div \frac{1}{2}$ (since $.5 = \frac{1}{2}$) $= \frac{7}{\not 8_4} \times \frac{\not 2}{1} = \frac{7}{4} = 1\frac{3}{4}$, or 1.75. Ans.

This can also be solved by reducing $\frac{7}{8}$ to its equivalent decimal and dividing by .5; $\frac{7}{8} = .875$; $.875 \div .5 = 1.75$. Since there are 3 decimal places in the dividend and 1 in the divisor, there are $3 - 1$ or 2 decimal places in the quotient.

(*c*) $\dfrac{.375 \times \frac{1}{4}}{\frac{5}{16} - .125} = ?$ We shall solve this problem by first reducing the decimals to their equivalent common fractions.

$.375 = \frac{375}{1000} = \frac{75}{200} = \frac{15}{40} = \frac{3}{8}$. $\frac{3}{8} \times \frac{1}{4} = \frac{3}{32}$, or the value of the numerator of the fraction.

$.125 = \frac{125}{1000} = \frac{25}{200} = \frac{1}{8}$. Reducing $\frac{1}{8}$ to 16ths, we have $\frac{1 \times 2}{8 \times 2} = \frac{2}{16}$. Then, $\frac{5}{16} - \frac{2}{16} = \frac{3}{16}$, or the value of the denominator of the fraction. The problem is now reduced to $\dfrac{\frac{3}{32}}{\frac{3}{16}} = ?$ $\dfrac{\frac{3}{32}}{\frac{3}{16}} = \frac{3}{32} \div \frac{3}{16} = \frac{\not 3}{\not 32_2} \times \frac{\not 16}{\not 3} = \frac{1}{2}$ or .5. Ans.

(74) $\dfrac{1.25 \times 20 \times 3}{87 + (11 \times 8)} = ?$ $\overline{459 + 32}$ In this problem $1.25 \times 20 \times 3$ constitutes the numerator of the complex fraction.

1.2 5

× 2 0

2 5.0 0

× 3

7 5

Multiplying the factors of the numerator together, we find their product to be 75.

The fraction $\frac{87+(11\times 8)}{459+32}$ constitutes the denominator of the complex fraction. The value of the numerator of this fraction equals $87+88 = 175$.

The numerator is combined as though it were written $87+(11\times 8)$, and its result is

$$\begin{array}{r} 11 \\ \times\ 8 \\ \hline 88 \\ +\ 87 \\ \hline 175 \end{array}$$

The value of the denominator of this fraction is equal to $459+32 = 491$. The problem then becomes

$$\frac{75}{\frac{175}{491}} = \frac{75}{1}\div\frac{175}{491} = \frac{75}{1}\times\frac{491}{175} = \frac{\overset{3}{\cancel{75}}\times 491}{\underset{7}{\cancel{175}}} = \frac{1473}{7} = 210\tfrac{3}{7}.$$ Ans.

(75) 1 plus .001 = 1.001. .01 plus .000001 = .010001. And $1.001-.010001 = .990999$. Ans.

ARITHMETIC.

(QUESTIONS 1-92. SEC. 2.)

(1) A certain per cent. of a number means so many hundredths of that number.

25% of 8,428 lb. means 25 hundredths of 8,428 lb. Hence, 25% of 8,428 lb. = .25 × 8,428 lb. = 2,107 lb. Ans.

(2) Here \$100 is the base and 1% = .01 is the rate. Then, .01 × \$100 = \$1. Ans.

(3) ½% means one-half of 1 per cent. Since 1% is .01, ½% is .005, for 2).0 1 0. And .005 × \$35,000 = \$175. Ans.
.0 0 5

(4) Here 50 is the base, 2 is the percentage, and it is required to find the rate. Applying rule, Art. **12,**

rate = percentage ÷ base;
rate = 2 ÷ 50 = .04, or 4%. Ans.

(5) By Art. **12,** rate = percentage ÷ base.*

As percentage = 10 and base = 10, we have

rate = 10 ÷ 10 = 1 = 100%.

Hence, 10 is 100% of 10. Ans.

* Remember that an expression of this form means that the first term is to be *divided by* the second term. Thus, as above, it means percentage *divided by* base.

(6) (*a*) Rate = percentage ÷ base. Art. **12.**

As percentage = \$176.54 and base = \$2,522, we have

rate = 176.54 ÷ 2,522 = .07 = 7%. Ans.

2 5 2 2) 1 7 6.5 4

.0 7

(*b*) Base = percentage ÷ rate. Art. **11.**

As percentage = 16.96 and rate = 8% = .08, we have

base = 16.96 ÷ .08 = 212. Ans.

.0 8) 1 6.9 6

2 1 2

(*c*) Amount is the sum of the base and percentage; hence, the percentage = amount minus the base.

Amount = 216.7025 and base = 213.5; hence, percentage = 216.7025 − 213.5 = 3.2025.

Rate = percentage ÷ base. Art. **12.**

Therefore, rate = 3.2025 ÷ 213.5 = .015 = 1½%. Ans.

2 1 3.5) 3.2 0 2 5 (.0 1 5 = 1½%.

2 1 3 5

1 0 6 7 5

1 0 6 7 5

(*d*) The difference is the remainder found by subtracting the percentage from the base; hence, base minus the difference = the percentage. Base = 207 and difference = 201.825; hence, percentage = 207 − 201.825 = 5.175.

Rate = percentage ÷ base. Art. **12.**

Therefore, rate = 5.175 ÷ 207 = .025 = .02½ = 2½%. Ans.

2 0 7) 5.1 7 5 (.0 2 5 = 2½%.

4 1 4

1 0 3 5

1 0 3 5

(7) In this problem \$5,500 is the amount, since it equals what he paid for the farm plus what he gained; 15% is the rate, and the cost (to be found) is the base. Applying rule, Art. **16,**

base = amount ÷ (1 + rate); hence,
base = \$5,500 ÷ (1 + .15) = \$4,782.61. Ans.

```
1.1 5 ) 5 5 0 0.0 0 0 0 ( 4 7 8 2.6 1—
        4 6 0
        -----
          9 0 0
          8 0 5
          -----
            9 5 0
            9 2 0
            -----
              3 0 0
              2 3 0
              -----
                7 0 0
                6 9 0
                -----
                  1 0 0
                  1 1 5
```

The example can also be solved as follows: 100% = cost; if he gained 15%, then 100 + 15 = 115% = \$5,500, the selling price.

If 115% = \$5,500, 1% = $\frac{1}{115}$ of \$5,500 = \$47.8261, and 100%, or the cost = 100 × \$47.8261 = \$4,782.61. Ans.

(8) 24 % of \$950 = 950 × .24 = \$228.00
12½% of \$950 = 950 × .125 = 118.75
17 % of \$950 = 950 × .17 = 161.50
53½% of \$950 = \$508.25

The total amount of his yearly expenses, then, is \$508.25; hence, his savings are \$950 − \$508.25 = \$441.75. Ans.

Or, as above, 24% + 12½% + 17% = 53½%, the total percentage of expenditures; hence, 100% − 53½% = 46½% = per cent. saved. And \$950 × .465 = \$441.75 = his yearly savings. Ans.

(9) The percentage is 961.38, and the rate is .37½. By Art. **11**,

base = percentage ÷ rate
= 961.38 ÷ .375 = 2,563.68, the number. Ans.

.3 7 5) 9 6 1.3 8 0 0 0 (2 5 6 3.6 8
7 5 0
2 1 1 3
1 8 7 5
2 3 8 8
2 2 5 0
1 3 8 0
1 1 2 5
2 5 5 0
2 2 5 0
3 0 0 0
3 0 0 0

Another method of solving is the following:

If $37\frac{1}{2}\%$ of a number is 961.38, then $.37\frac{1}{2}$ times the number = 961.38, and the number = $961.38 \div .37\frac{1}{2}$, which, as above = 2,563.68. Ans.

(10) Here \$1,125 is 30% of some number; hence, \$1,125 = the percentage, 30% = the rate, and the required number is the base. Applying rule, Art. **11,**

base = percentage ÷ rate = \$1,125 ÷ .30 = \$3,750.

Since \$3,750 is $\frac{3}{4}$ of the property, one of the fourths is $\frac{1}{3}$ of \$3,750 = \$1,250, and 4 fourths, or the entire property, is 4 × \$1,250 = \$5,000. Ans.

(11) Here \$4,810 is the difference and 35% the rate. By Art. **17,**

base = difference ÷ (1 − rate)

= \$4,810 ÷ (1 − .35) = \$4,810 ÷ .65 = \$7,400. Ans.

.6 5) 4 8 1 0.0 0 (7 4 0 0
4 5 5
2 6 0
2 6 0

1.0 0
.3 5
.6 5

Solution can also be effected as follows: 100% = the sum diminished by 35%, then (1 − .35) = .65, which is \$4,810.

If 65% = \$4,810, 1% = $\frac{1}{65}$ of \$4,810 = \$74, and 100% = 100 × \$74 = \$7,400. Ans.

(12) In this example the sales on Monday amounted to \$197.55, which was $12\frac{1}{2}\%$ of the sales for the entire week; i.e., we have given the percentage, \$197.55, and the rate, $12\frac{1}{2}\%$,

and the required number (or the amount of sales for the week) equals the base. By Art. **11,**

base = percentage ÷ rate = \$197.55 ÷ .125;

or,

```
.1 2 5 ) 1 9 7.5 5 0 0 ( 1 5 8 0.4
         1 2 5
         -----
           7 2 5
           6 2 5
           -----
           1 0 0 5
           1 0 0 0
           -------
               5 0 0
               5 0 0
               -----
```

Therefore, base = \$1,580.40, which also equals the sales for the week. Ans.

(13) 16.5 miles = $12\frac{1}{2}\%$ of the entire length of the road. We wish to find the *entire* length.

16.5 miles is the percentage, $12\frac{1}{2}\%$ is the rate, and the entire length will be the base. By Art. **11,**

base = percentage ÷ rate = $16.5 \div .12\frac{1}{2}$.

```
.1 2 5 ) 1 6.5 0 0 ( 1 3 2 miles.  Ans.
         1 2 5
         -----
           4 0 0
           3 7 5
           -----
             2 5 0
             2 5 0
             -----
```

(14) Here we have given the difference, or \$35, and the rate, or 60%, to find the base. We use the rule in Art. **17,**

base = difference ÷ (1 − rate)

= \$35 ÷ (1 − .60) = \$35 ÷ .40 = \$87.50. Ans.

```
.4 0 ) 3 5.0 0 0 ( 8 7.5
       3 2 0
       -----
         3 0 0
         2 8 0
         -----
           2 0 0
           2 0 0
           -----
```

Or, 100% = whole debt; 100% − 60% = 40% = $35.

If 40% = $35, then 1% = $\frac{1}{40}$ of $35 = $\frac{\$35}{40}$, and 100% = $\frac{\$35}{40} \times 100 = \$87.50$. Ans.

(15) 28 rd. 4 yd. 2 ft. 10 in. to inches.

$\times 5\frac{1}{2}$

154

+ 4

158 yards.

× 3

474

+ 2

476 feet.

× 12

5712

+ 10

5722 inches.

Since there are $5\frac{1}{2}$ yards in 1 rod, in 28 rods there are 28 $\times 5\frac{1}{2}$, or 154 yards; 154 yards + 4 yards = 158 yards. There are 3 feet in 1 yard; therefore, in 158 yards there are 3 × 158, or 474 feet; 474 feet + 2 feet = 476 feet. There are 12 inches in 1 foot, and in 476 feet there are 12 × 476, or 5,712 inches; 5,712 inches + 10 inches = 5,722 inches. Ans.

(16)

12)5722 inches.

3)476 + 10 inches.

$5\frac{1}{2}$)158 + 2 feet.

28 + 4 yards.

28 rd. 4 yd. 2 ft. 10 in. Ans.

Explanation.—There are 12 inches in 1 foot; hence, in 5,722 inches there are as many feet as 12 is contained times in 5,722 inches, or 476 feet and 10 inches remaining. Write these 10 inches as a remainder. There are 3 feet in 1 yard; hence, in 476 feet there are as many yards as 3 is contained times in 476 feet, or 158 yards and 2 feet remaining. There are $5\frac{1}{2}$ yards in 1 rod; hence, in 158 yards there are 28 rods and 4 yards remaining. Then, in 5,722 inches, there are 28 rd. 4 yd. 2 ft. 10 in.

(17)

5 weeks 3.5 days.

× 7

35 days in 5 weeks.

+ 3.5

38.5 days.

Then, we find how many seconds there are in 38.5 days.

38.5 days.
× 24 hours in 1 day.
1540
770
924.0 hours in 38.5 days.
× 60 minutes in 1 hour.
55440 minutes in 38.5 days.
× 60 seconds in 1 minute.
3326400 seconds in 38.5 days. Ans.

(18) Since there are 24 gr. in 1 pwt., in 13,750 gr. there are as many pennyweights as 24 is contained times in 13,750, or 572 pwt. and 22 gr. remaining. Since there are 20 pwt. in 1 oz., in 572 pwt. there are as many ounces as 20 is contained times in 572, or 28 oz. and 12 pwt. remaining. Since there are 12 oz. in 1 lb. (Troy), in 28 oz. there are as many pounds as 12 is contained times in 28, or 2 lb. and 4 oz. remaining. We now have the pounds and ounces required by the problem; therefore, in 13,750 gr. there are 2 lb. 4 oz. 12 pwt. 22 gr.

24)13750 gr.
20)572 pwt. + 22 gr.
12)28 oz. + 12 pwt.
2 lb. + 4 oz.

2 lb. 4 oz. 12 pwt. 22 gr. Ans.

(19) 100)4763254 li.
80)47632 + 54 li.
595 + 32 ch.

595 mi. 32 ch. 54 li. Ans.

EXPLANATION.—There are 100 links in 1 chain; hence, in 4,763,254 li. there are as many chains as 100 is contained times in 4,763,254 li., or 47,632 ch. and 54 li. remaining. Write the 54 li. as a remainder. There are 80 ch. in 1 mile; hence, in 47,632 ch. there are as many miles as 80

is contained times in 47,632 ch., or 595 mi. and 32 ch. remaining.

Then, in 4,763,254 li. there are 595 mi. 32 ch. 54 li.

(20) $1\,7\,2\,8\overline{)\,7\,6\,4\,3\,2\,5}$ cu. in.

$2\,7\overline{)\,4\,4\,2} + 549$ cu. in.

$1\,6$ cu. yd. $+ 10$ cu. ft.

16 cu. yd. 10 cu. ft. 549 cu. in. Ans.

EXPLANATION.—There are 1,728 cu. in. in 1 cubic foot; hence, in 764,325 cu. in. there are as many cubic feet as 1,728 is contained times in 764,325, or 442 cu. ft. and 549 cu. in. remaining. Write the 549 cu. in. as a remainder. There are 27 cu. ft. in 1 cubic yard; hence, in 442 cu. ft. there are as many cubic yards as 27 is contained times in 442 cu. ft., or 16 cu. yd. and 10 cu. ft. remaining. Then, in 764,325 cu. in. there are 16 cu. yd. 10 cu. ft. 549 cu. in.

(21) We must arrange the different terms in columns, taking care to have like denominations in the same column.

rd.	yd.	ft.	in.
2	2	2	3
	4	1	9
		2	7
3	$2\frac{1}{2}$	0	7

or, 3 rd. 2 yd. 2 ft. 1 in. Ans.

EXPLANATION.—We begin to add at the right-hand column. $7+9+3 = 19$ in.; as 12 in. make 1 foot, 19 in. = 1 ft. and 7 in. Place the 7 in. in the inches column, and reserve the 1 ft. to add to the next column.

1 (reserved) $+2+1+2 = 6$ ft. Since 3 ft. make 1 yard, 6 ft. = 2 yd. and 0 ft. remaining. Place the cipher in the column of feet and reserve the 2 yd. for the next column.

2 (reserved) $+4+2 = 8$ yd. Since $5\frac{1}{2}$ yd. = 1 rd., 8 yd. = 1 rd. and $2\frac{1}{2}$ yd. Place $2\frac{1}{2}$ yd. in the yards column and reserve 1 rd. for the next column; 1 (reserved) $+2 = 3$ rd.

Ans. = 3 rd. $2\frac{1}{2}$ yd. 0 ft. 7 in.,
or 3 rd. 2 yd. 1 ft. 13 in.,
or 3 rd. 2 yd. 2 ft. 1 in.

(22) We write the compound numbers so that the units of the same denomination shall stand in the same column.

gal.	qt.	pt.	gi.
3	3	1	3
6	0	1	2
4	0	0	1
	8	5	0
16 gal.	3 qt.	0 pt.	2 gi.

Beginning to add with the lowest denomination, we find that the sum of the gills is $1+2+3=6$. Since there are 4 gi. in 1 pint, in 6 gi. there are as many pints as 4 is contained times in 6, or 1 pt. and 2 gi. We place 2 gi. under the gills column and reserve the 1 pt. for the pints column; the sum of the pints is 1 (reserved) $+5+1+1=8$. Since there are 2 pt. in 1 quart, in 8 pt. there are as many quarts as 2 is contained times in 8, or 4 qt. and 0 pt. We place the cipher under the column of pints and reserve the 4 for the quarts column. The sum of the quarts is 4 (reserved) $+8+3=15$. Since there are 4 qt. in 1 gallon, in 15 qt. there are as many gallons as 4 is contained times in 15, or 3 gal. and 3 qt. remaining. We now place the 3 under the quarts column and reserve the 3 gal. for the gallons column. The sum of the gallons column is 3 (reserved) $+4+6+3=16$ gal. Since we cannot reduce 16 gal. to any higher denomination, we have 16 gal. 3 qt. 0 pt. and 2 gi. for the answer.

(23) Reduce the grains, pennyweights, and ounces to higher denominations.

2 4) 2 4 0 gr. 2 0) 1 2 5 pwt. 1 2) 5 0 oz.

1 0 pwt. 6 oz. 5 pwt. 4 lb. 2 oz.

Then, 3 lb. + 4 lb. 2 oz. + 6 oz. 5 pwt. + 10 pwt. =

lb.	oz.	pwt.
3		
4	2	
	6	5
		10
7 lb.	8 oz.	15 pwt. Ans.

(24) Since "seconds" is the lowest denomination in this problem, we find their sum first, which is $11+29+25+30$

deg.	min.	sec.
11	16	12
13	19	30
20	0	25
0	26	29
10	17	11
55°	19′	47″

+12, or 107 seconds. Since there are 60 seconds in 1 minute, in 107 seconds there are as many minutes as 60 is contained times in 107, or 1 minute and 47 seconds remaining. We place the 47 under the seconds column and reserve the 1 for the minutes column. The sum of the minutes is 1 (reserved) +17+26+19+16, or 79. Since there are 60 minutes in 1 degree, in 79 minutes there are as many degrees as 60 is contained times in 79, or 1 degree and 19 minutes remaining. We place the 19 under the minutes column and reserve the 1 degree for the degrees column. The sum of the degrees is 1 (reserved) +10+20+13+11, or 55 degrees. Since we cannot reduce 55 degrees to any higher denomination, we have 55° 19′ 47″ for the answer.

(25) Since "inches" is the lowest denomination in this problem, we find their sum first, which is 11+8+6, or 25 inches. Since there are 12 inches in 1 foot, in 25 inches there are as many feet as 12 is contained times in 25, or 2 feet and 1 inch remaining. Place the 1 inch under the inches column, and reserve the 2 feet to add to the column of feet. The sum of the feet is 2 feet (reserved) +2+1 = 5 feet. Since there are 3 feet in 1 yard, in 5 feet there are as many yards as 3 is contained times in 5 feet, or 1 yard and 2 feet remaining. Place the 2 feet under the column of feet, and reserve the 1 yard to add to the column of yards. The sum of the yards is 1 yard (reserved) +4 +5 = 10 yards. Since there are $5\frac{1}{2}$ yards in 1 rod, in 10 yards there are as many rods as $5\frac{1}{2}$ is contained times in 10, or 1 rod and $4\frac{1}{2}$ yards remaining. Place the $4\frac{1}{2}$ yards under

rd.	yd.	ft.	in.
130	5	1	6
215	0	2	8
304	4	0	11
650	$4\frac{1}{2}$	2	1

mi.	rd.	yd.	ft.	in.	
or, 2	10	5	0	7	Ans.

the column of yards, and reserve the 1 rod for the column of rods. The sum of the rods is 1 (reserved) $+304+215+130 = 650$ rods. Place 650 rods under the column of rods. Therefore, the sum is 650 rd. $4\frac{1}{2}$ yd. 2 ft. 1 in. Or, since $\frac{1}{2}$ a yard = 1 ft. 6 in., and since there are 320 rods in 1 mile, the sum may be expressed as 2 mi. 10 rd. 5 yd. 0 ft. 7 in. Ans.

(26) Since "square links" is the lowest denomination in this problem, we find their sum first, which is $21+23+16+18+23+21$, or 122 square links. Place 122 square links under the column of square links. The sum of the square rods is $2+3+2+2+2+3$, or 14 square rods. Place 14 square rods under the column of square rods. The sum of the square chains is 323 square chains. Since there are 10 square chains in 1 acre, in 323 square chains there are as many acres as 10 is contained times in 323 square chains, or 32 acres and 3 square chains remaining. Place 3 square chains under the column of square chains, and reserve the 32 acres to add to the column of acres. The sum of the acres is 32 acres (reserved) $+46+25+56+47+28+21$, or 255 acres. Place 255 acres under the column of acres. Therefore, the sum is 255 A. 3 sq. ch. 14 sq. rd. 122 sq. li. Ans.

A.	sq. ch.	sq. rd.	sq. li.
21	67	3	21
28	78	2	23
47	6	2	18
56	59	2	16
25	38	3	23
46	75	2	21
255	3	14	122

(27) Before we can subtract 300 ft. from 20 rd. 2 yd. 2 ft. and 9 in., we must reduce the 300 ft. to higher denominations.

Since there are 3 feet in 1 yard, in 300 feet there are as many yards as 3 is contained times in 300, or 100 yards. There are $5\frac{1}{2}$ yards in 1 rod, hence in 100 yards there are as many rods as $5\frac{1}{2}$, or $\frac{11}{2}$, is contained times in $100 = 18\frac{2}{11}$ rods.

$$100 \div 5\tfrac{1}{2} = 100 \times \tfrac{2}{11} = \frac{100 \times 2}{11} = \frac{200}{11}$$

$$11\overline{)\,2\,0\,0\,(\,18\tfrac{2}{11}\text{ rd.}}$$

$$\begin{array}{r} 11 \\ \hline 90 \\ 88 \\ \hline 2 \end{array}$$

Since there are $5\frac{1}{2}$ or $\frac{11}{2}$ yards in 1 rod, in $\frac{2}{11}$ rod there are $\frac{2}{11} \times \frac{11}{2}$, or 1 yard, so we find that 300 feet equals 18 rods and 1 yard. The problem now is as follows: From 20 rd. 2 yd. 2 ft. and 9 in. take 18 rd. and 1 yd.

We place the smaller number under the larger one, so that the units of the same denomination fall in the same column. Beginning with the lowest denomination, we see that 0 inches from 9 inches leaves 9 inches. Going to the next higher denomination, we see that 0 feet from 2 feet leaves 2 feet. Substracting 1 yard from 2 yards, we have 1 yard remaining, and 18 rods from 20 rods leaves 2 rods. Therefore, the difference is 2 rd. 1 yd. 2 ft. 9 in. Ans.

rd.	yd.	ft.	in.
20	2	2	9
18	1	0	0
2	1	2	9

(28)

A.	sq. rd.	sq. yd.	
114	80	25	
75	70	30	
39	9	$25\frac{1}{4}$	Ans.

Explanation.—Place the subtrahend under the minuend so that like denominations are under each other. Then begin at the right with the lowest denomination. We cannot subtract 30 from 25, so we take 1 square rod (= $30\frac{1}{4}$ square yards) from 80 square rods, leaving 79 square rods; adding $30\frac{1}{4}$ square yards to 25 square yards, we have $55\frac{1}{4}$ square yards; subtracting 30 from $55\frac{1}{4}$ square yards, leaves $25\frac{1}{4}$ yards; we now subtract 70 square rods from 79 square rods, which leaves 9 square rods; next, we subtract 75 acres from

114 acres, which leaves 39 acres, which we place under the column of acres.

(29) If 10 gal. 2 qt. and 1 pt. of molasses are sold from a hogshead at one time, and 26 gal. 3 qt. are sold at another time, then the total amount of molasses sold equals 10 gal. 2 qt. 1 pt. plus 26 gal. 3 qt.

Since the pint is the lowest denomination, we add the pints first, which equal $0+1$, or 1 pint. We cannot reduce 1 pint to any higher denomination, so we place it under the pints column. The number of quarts is $3+2$, or 5. Since there are 4 quarts in 1 gallon, in 5 quarts there are as many gallons as 4 is contained times in 5, or 1 gallon and 1 quart remaining. We place the 1 quart under the quarts column, and reserve the 1 gallon to add to the column of gallons. The number of gallons equals 1 (reserved) $+26+10$, or 37 gallons.

gal.	qt.	pt.
10	2	1
26	3	0
37	1	1

If 37 gal. 1 qt. and 1 pt. are sold from a hogshead of molasses (63 gal.), there remains the difference between 63 gal. and 37 gal. 1 qt. 1 pt., or 25 gal. 2 qt. 1 pt.

63 gal. is the same as 62 gal. 3 qt. 2 pt., since 1 gal. equals 4 qt., and 1 qt. equals 2 pt.

Beginning with the lowest denomination, we subtract 1 pt. from the 2 pt. 1 pint from 2 pints leaves 1 pint, 1 quart from 3 quarts leaves 2 quarts, and 37 gallons from 62 gallons leaves 25 gallons. Therefore, there are 25 gal. 2 qt. and 1 pt. of molasses remaining in the hogshead. Ans.

gal.	qt.	pt.
62	3	2
37	1	1
25	2	1

(30) If a person were born June 19, 1850, in order to find how old he would be on Aug. 3, 1892, subtract the earlier date from the later date.

On Aug. 3, 7 mo. and 3 da. have elapsed from the beginning of the year, and on June 19, 5 mo. and 19 da.

Beginning with the lowest denomination, we find that 19

days cannot be taken from 3 days, so we take 1 month from 7 months. The 1 month which we took equals 30 days, for in all cases 30 days are allowed to a month. Adding 30 days to the 3 days, we have 33 days; subtracting 19 days from 33 days, we have 14 days remaining. Since we borrowed 1 month from the months column, we have $7-1$, or 6 months remaining; subtracting 5 months from 6 months, we have 1 month remaining. 1,850 from 1,892 leaves 42 years. Therefore, he would be 42 years, 1 month, and 14 days old. Ans.

yr.	mo.	da.
1892	7	3
1850	5	19
42	1	14

(31) If a note given Aug. 5, 1890, were paid June 3, 1892, in order to find the length of time it was due, subtract the earlier date from the later date.

Beginning with the lowest denomination, we find that 5 cannot be subtracted from 3, so we take a unit from the next higher denomination, which is months. The 1 month which we take equals 30 days. Adding the 30 days to the 3 days, we have 33 days. 5 days from 33 days leaves 28 days. Since we took 1 month from the months column, only 4 months remain. 7 months cannot be taken from 4 months, so we take 1 year from the years column, which equals 12 months. 12 months $+4$ months $=$ 16 months. 7 months from 16 months $=$ 9 months. Since we took 1 year from the years column, we have $1,892-1$, or 1,891 remaining. 1,890 from 1,891 leaves 1 year. Hence, the note ran 1 year, 9 months, and 28 days. Ans.

yr.	mo.	da.
1892	5	3
1890	7	5
1	9	28

(32) Write the number of the year, month, day, hour, and minute of the earlier date under the year, month, day, hour, and minute of the later date, and subtract.

22 minutes before 8 o'clock is the same as 38 minutes after 7 o'clock. 7 o'clock P. M. is 19 hours from the beginning of the day, as there are 12 hours in the morning and 7 in the

afternoon. December is 11 months from the beginning of the year.

10 o'clock A. M. is 10 hours from the beginning of the day. July is 6 months from the beginning of the year. The minuend would be the later date, or 1,888 years, 11 months, 11 days, 19 hours, and 38 minutes.

The subtrahend would be the earlier date or 1,883 years, 6 months, 3 days, 10 hours, and 16 minutes.

Subtracting, we have

yr.	mo.	da.	hr.	min.
1888	11	11	19	38
1883	6	3	10	16
5	5	8	9	22

or, 5 yr. 5 mo. 8 da. 9 hr. and 22 min. Ans.

16 minutes subtracted from 38 minutes leaves 22 minutes; 10 hours from 19 hours leaves 9 hours; 3 days from 11 days leaves 8 days; 6 months subtracted from 11 months leaves 5 months; 1,883 from 1,888 leaves 5 years.

(33) In multiplication of denominate numbers, we place the multiplier under the lowest denomination of the multiplicand, as

$$\begin{array}{r} 17 \text{ ft. } 3 \text{ in.} \\ \times\quad 51 \\ \hline 879 \text{ ft. } 9 \text{ in.} \end{array}$$

and begin at the right to multiply. $51 \times 3 = 153$ in. As there are 12 inches in 1 foot, in 153 in. there are as many feet as 12 is contained times in 153, or 12 feet and 9 inches remaining. Place the 9 inches under the inches, and reserve the 12 feet. 51×17 ft. $= 867$ ft. 867 ft. $+ 12$ ft. (reserved) $=$ 879 ft.

879 feet can be reduced to higher denominations by dividing by 3 feet to find the number of yards, and by $5\frac{1}{2}$ yards to find the number of rods.

$$\begin{array}{r} 3\,)\,879 \text{ ft. } 9 \text{ in.} \\ 5.5\,)\,293 \text{ yd.} \\ 53 \text{ rd. } 1\tfrac{1}{2} \text{ yd.} \end{array}$$

Then, product = 53 rd. $1\frac{1}{2}$ yd. 0 ft. 9 in., or 53 rd. 1 yd. 2 ft. 3 in. Ans.

(34)

	qt.	pt.	gi.	
	3	1	3	
			4.7	
	18.2	0	.1	
or,	18 qt.	0 pt.	1.7 gi.	
or, 4 gal.	2 qt.	0 pt.	1.7 gi.	Ans.

Place the multiplier under the lowest denomination of the multiplicand, and proceed to multiply. 4.7×3 gi. = 14.1 gi. As 4 gi. = 1 pt., there are as many pints in 14.1 gi. as 4 is contained times in 14.1 = 3.5 pt. and .1 gi. over. Place .1 under gills and carry the 3.5 pt. forward. 4.7×1 pt. = 4.7 pt.; 4.7 + 3.5 pt. = 8.2 pt. As 2 pt. = 1 qt., there are as many quarts in 8.2 pt. as 2 is contained times in 8.2 = 4.1 qt. and no pints over. Place a cipher under the pints, and carry the 4.1 qt. to the next product. 4.7×3 qt. = 14.1; 14.1 + 4.1 qt. = 18.2 qt. The answer now is 18.2 qt. 0 pt. .1 gi. Reducing the fractional part of a quart, we have 18 qt. 0 pt. 1.7 gi. (.2 qt. = $.2 \times 8$ = 1.6 gi.; 1.6 + .1 gi. = 1.7 gi.). Then we can reduce 18 qt. to gallons (18 ÷ 4 = 4 gal. and 2 qt.) = 4 gal. 2 qt. 1.7 gi. Ans.

The answer may be obtained in another and much easier way by reducing all to gills, multiplying by 4.7, and then changing back to quarts and pints. Thus,

3 qt.
× 2 pt.
6 pt.
+ 1 pt.
7 pt.
× 4 gi.
28 gi.
+ 3 gi.
31 gi.

3 qt. 1 pt. 3 gi. = 31 gi.
31 gi. × 4.7 = 145.7 gi.

4) 145.7 gi.
2) 36 pt. + 1.7 gi.
18 qt. + 0 pt.

Ans. = 18 qt. 1.7 gi.;
or, 4 gal. 2 qt. 1.7 gi.

(35) (3 lb. 10 oz. 13 pwt. 12 gr.) × 1.5 = ?

3 lb. 10 oz. 13 pwt. 12 gr.
× 1 2
3 6 oz.
+ 1 0
4 6 oz.
× 2 0
9 2 0 pwt.
+ 1 3
9 3 3 pwt.
× 2 4
2 2 3 9 2 gr.
+ 1 2
2 2 4 0 4 gr.

22,404 gr. × 1.5 = 33,606 gr.

2 4) 3 3 6 0 6 gr.
2 0) 1 4 0 0 pwt. + 6 gr.
1 2) 7 0 oz. + 0 pwt.
5 lb. + 10 oz.

Since there are 24 gr. in 1 pwt., in 33,606 gr. there are as many pennyweights as 24 is contained times in 33,606, or 1,400 pwt. and 6 gr. remaining. This gives us the number of grains in the answer. We now reduce 1,400 pwt. to higher denominations. Since there are 20 pwt. in 1 oz., in 1,400 pwt. there are as many ounces as 20 is contained times in 1,400, or 70 oz. and 0 pwt. remaining; therefore, there are 0 pwt. in the answer. We reduce 70 oz. to higher denominations. Since there are 12 oz. in 1 lb., in 70 oz. there are as many pounds as 12 is contained times in 70, or 5 lb. and 10 oz. remaining. We cannot reduce 5 lb. to any higher denominations. Therefore, our answer is 5 lb. 10 oz. 6 gr.

Another but more complicated way of working this problem is as follows:

lb.	oz.	pwt.	gr.	
3	10	13	12	
			1.5	
4.5	15	19.5	18	
or, 4	21	19	30	
or, 5	10	0	6	Ans.

To get rid of the decimal in the pounds, reduce .5 of a pound to ounces. Since 1 lb. = 12 oz., .5 of a pound equals .5 lb. × 12 = 6 oz. 6 oz. + 15 oz. = 21 oz. We now have 4 lb. 21 oz. 19.5 pwt. and 18 gr.; but we still have a decimal in the column of pwt., so we reduce .5 pwt. to grains to get rid of it. Since 1 pwt. = 24 gr., .5 pwt. = .5 pwt. × 24 = 12 gr. 12 gr. + 18 gr. = 30 gr. We now have 4 lb. 21 oz. 19 pwt. and 30 gr. Since there are 24 gr. in 1 pwt., in 30 gr. there is 1 pwt. and 6 gr. remaining. Place 6 gr. under the column of grains and add 1 pwt. to the pwt. column. Adding 1 pwt., we have 19 + 1 = 20 pwt. Since there are 20 pwt. in 1 oz., we have 1 oz. and 0 pwt. remaining. Write the 0 pwt. under the pwt. column, and reserve the 1 oz. to the oz. column. 21 oz. + 1 oz. = 22 oz. Since there are 12 oz. in 1 lb., in 22 oz. there is 1 lb. and 10 oz. remaining. Write the 10 oz. under the ounce column, and reserve the 1 lb. to add to the pounds column. 4 lb. + 1 lb. (reserved) = 5 lb. Hence, the answer equals 5 lb. 10 oz. 6 gr.

(36) . If each barrel of apples contains 2 bu. 3 pk. and 6 qt., then 9 bbl. will contain 9 × (2 bu. 3 pk. 6 qt.).

We write the multiplier under the lowest denomination of the multiplicand, which is quarts in this problem. 9 times 6 qt. equals 54 qt. There are 8 qt. in 1 pk., and in 54 qt. there are as many pecks as 8 is contained times in 54, or 6 pk. and 6 qt. We write the 6 qt. under the column of quarts, and reserve the 6 pk. to add to the product of the pecks. 9 times 3 pk. equals 27 pk.; 27 pk. plus the 6 pk. (reserved) equals 33 pk. Since there are 4 pk. in 1 bu., in 33 pk. there are as many bushels as 4 is contained times in 33, or 8 bu. and 1 pk. remaining. We write the 1 pk. under the column of pecks, and reserve the 8 bu.

bu.	pk.	qt.
2	3	6
		9
18	27	54
or, 26	1	6

for the product of the bushels. 9 times 2 bu. plus the 8 bu. (reserved) equals 26 bu. Therefore, we find that 9 bbl. contain 26 bu. 1 pk. 6 qt. of apples. Ans.

(37) (7 T. 15 cwt. 10.5 lb.) × 1.7 = ? When the multiplier is a decimal, instead of multiplying the denominate numbers as in the case when the multiplier is a whole number, it is much easier to reduce the denominate numbers to the lowest denomination given; then multiply that result by the decimal, and, lastly, reduce the product to higher denominations. Although the correct answer can be obtained by working examples involving decimals in the manner as in the last example, it is much more complicated than this method.

7 T. 15 cwt. 10.5 lb.
× 2 0
1 4 0 cwt.
+ 1 5
1 5 5 cwt.
× 1 0 0
1 5 5 0 0 lb.
+ 1 0.5
1 5 5 1 0.5 lb.

15,510.5 lb. × 1.7 = 26,367.85 lb.

There are 100 lb. in 1 cwt., and in 26,367.85 lb. there are as many cwt. as 100 is contained times in 26,367.85, which equals 263 cwt. and 67.85 lb. remaining. Since we have the number of pounds for our answer, we reduce 263 cwt. to higher denominations. There are 20 cwt. in 1 ton, and in 263 cwt. there are as many tons as 20 is contained times in 263, or 13 T. and 3 cwt. remaining. Since we cannot reduce 13 T. any higher, our answer is 13 T. 3 cwt. 67.85 lb. Or, since .85 lb. = .85 lb. × 16 = 13.6 oz., the answer may be written 13 T. 3 cwt. 67 lb. 13.6 oz.

1 0 0) 2 6 3 6 7.8 5 lb.
2 0) 2 6 3 cwt. + 67.85 lb.
1 3 T. + 3 cwt.

(38) 7) 3 5 8 A. 5 7 sq. rd. 6 sq. yd. 2 sq. ft.
5 1 A. 3 1 sq. rd. 0 sq. yd. 8 sq. ft. Ans.

We begin with the highest denomination, and divide each term in succession by 7.

7 is contained in 358 A. 51 times and 1 A. remaining. We write the 51 A. under the 358 A. and reduce the remaining 1 A. to square rods = 160 sq. rd.; 160 sq. rd. + the 57 sq. rd. in the dividend = 217 sq. rd. 7 is contained in 217 sq. rd. 31 times and 0 sq. rd. remaining. 7 is not contained in 6 sq. yd., so we write 0 under the sq. yd. and reduce 6 sq. yd. to square feet. 9 sq. ft. × 6 = 54 sq. ft. 54 sq. ft. + 2 sq. ft. in the dividend = 56 sq. ft. 7 is contained in 56 sq. ft. 8 times. We write 8 under the 2 sq. ft. in the dividend.

(39) 12) 282 bu. 3 pk. 1 qt. 1 pt.

23 bu. 2 pk. 2 qt. $\frac{1}{4}$ pt. Ans.

12 is contained in 282 bu. 23 times and 6 bu. remaining. We write 23 bu. under the 282 bu. in the dividend, and reduce the remaining 6 bu. to pecks = 24 pk. + the 3 pk. in the dividend = 27 pk. 12 is contained in 27 pk. 2 times and 3 pk. remaining. We write 2 pk. under the 3 pk. in the dividend, and reduce the remaining 3 pk. to quarts. 3 pk. = 24 qt.; 24 qt. + the 1 qt. in the dividend = 25 qt. 12 is contained in 25 qt. 2 times and 1 qt. remaining. We write 2 qt. under the 1 qt. in the dividend, and reduce 1 qt. to pints = 2 pt. + the 1 pt. in the dividend = 3 pt. $3 \div 12 = \frac{3}{12}$ or $\frac{1}{4}$ pt.

(40) We must first reduce 23 miles to feet before we can divide by 30 feet. 1 mile contains 5,280 ft.; hence 23 mi. contain $5{,}280 \times 23 = 121{,}440$ ft.

121,440 ft. ÷ 30 ft. = 4,048 rails for 1 side of the track.

The number of rails for 2 sides of the track = $2 \times 4{,}048$, or 8,096 rails. Ans.

(41) In this case, where both dividend and divisor are compound, reduce each to the lowest denomination mentioned in either, and then divide as in simple numbers.

1 bu. 1 pk. 7 qt.
× 4
4 pk.
+ 1
5 pk.
× 8
40 qt.
+ 7
47 qt.

3 5 6 bu. 3 pk. **5 qt.**
× 4
1 4 2 4 pk.
+ 3
1 4 2 7 pk.
× 8
1 1 4 1 6 qt.
+ 5
1 1 4 2 1 qt.

47)11421(243
94
202
188
141
141

11,421 qt. ÷ 47 qt. = 243 boxes. Ans.

(42) We must first reduce 16 square miles to acres.

In 1 sq. mi. there are 640 A., and in 16 sq. mi. there are 16×640 A. = 10,240 A.

62)10240 A.

165 A. 25 sq. rd. 24 sq. yd. 3 sq. ft. 80+ sq. in. Ans.

62 is contained in 10,240 A. 165 times and 10 A. remaining. We write 165 A. under the 10,240 A. in the dividend and reduce 10 A. to sq. rd. In 1 A. there are 160 sq. rd., and in 10 A. there are $10 \times 160 = 1{,}600$ sq. rd. 62 is contained in 1,600 sq. rd. 25 times and 50 sq. rd. remaining. We write 25 sq. rd. in the quotient and reduce 50 sq. rd. to sq. yd. In 1 sq. rd. there are $30\frac{1}{4}$ sq. yd., and in 50 sq. rd. there are $50 \times 30\frac{1}{4}$ sq. yd. = $1{,}512\frac{1}{2}$ sq. yd. 62 is contained in $1{,}512\frac{1}{2}$ sq. yd. 24 times and $24\frac{1}{4}$ sq. yd. remaining. We write 24 sq. yd. in the quotient and reduce $24\frac{1}{2}$ sq. yd. to sq. ft. In 1 sq. yd. there are 9 sq. ft., and in $24\frac{1}{2}$ sq. yd. there are $24\frac{1}{2} \times 9 = 220\frac{1}{2}$ sq. ft. 62 is contained in $220\frac{1}{2}$ sq. ft. 3 times and $34\frac{1}{2}$ sq. ft. remaining. We write 3 sq. ft. in the quotient and reduce $34\frac{1}{2}$ sq. ft. to sq. in. In 1 sq. ft. there are 144 sq. in., and in $34\frac{1}{2}$ sq. ft. there are $34\frac{1}{2} \times 144 = 4{,}968$ sq. in.

62 is contained in 4,968 sq. in. 80 times and 8 sq. in. remaining. We write 80+ sq. in. in the quotient.

It should be borne in mind that it is only for the purpose of illustrating the method that this problem is carried out to square inches. It is not customary to reduce any lower than square rods in calculating the area of a farm.

(43) To square a number, we must multiply the number by itself once; that is, use the number twice as a factor. Thus, the second power of 108 is $108 \times 108 = 11{,}664$. Ans.

$$\begin{array}{r} 108 \\ 108 \\ \hline 864 \\ 108 \\ \hline 11664 \end{array}$$

(44) The third power of 181.25 equals the number obtained by using 181.25 as a factor three times. Thus, the third power of 181.25 is $181.25 \times 181.25 \times 181.25 = 5{,}954{,}345.703125$. Ans.

$$\begin{array}{r} 181.25 \\ 181.25 \\ \hline 90625 \\ 36250 \\ 18125 \\ 1450000 \\ 18125 \\ \hline 32851.5625 \\ 181.25 \\ \hline 16425781250 \\ 6570312500 \\ 3285156250 \\ 262812500000 \\ 3285156250 \\ \hline 5954345.703125 \end{array}$$

Since there are 2 decimal places in the multiplier and 2 in the multiplicand, there are $2+2 = 4$ decimal places in the first product.

Since there are 4 decimal places in the multiplicand and 2 in the multiplier, there are $4+2 = 6$ decimal places in the final product.

(45) The fourth power of 27.61 is the number obtained by using 27.61 as a factor four times. Thus, the fourth power of 27.61 is $27.61 \times 27.61 \times 27.61 \times 27.61 = 581{,}119.73780641$. Ans.

```
      27.61
      27.61
      ----
       2761
     16566
   19327
   5522
   ------
   762.3121
       27.61
   -------
   7623121
 45738726
 5336184 7
1524624 2
---------
2104 7.437081
       27.61
   ---------
   2104743708 1
 126284622486
14733205956 7
4209487416 2
------------
581119.7378064 1
```

Since there are 2 decimal places in the multiplier and 2 in the multiplicand, there are $2+2=4$ decimal places in the first product.

Since there are 4 decimal places in the multiplicand and 2 in the multiplier, there are $4+2=6$ decimal places in the second product.

Since there are 6 decimal places in the multiplicand and 2 in the multiplier, there are $6+2=8$ decimal places in the final product.

(46) (*a*) $106^2 = 106 \times 106 = 11{,}236$. Ans.

```
  106
  106
  ---
  636
1060
----
11236
```

(*b*) $(182\frac{1}{8})^2 = 182\frac{1}{8} \times 182\frac{1}{8} = 33{,}169.515625$. Ans.

```
1/8 = 8 ) 1.000
          .125
```

```
   182.125
   182.125
   -------
   910625
  364250
 182125
 364250
1457000
182125
-------
33169.515625
```

Since there are 3 decimal places in the multiplier and 3 in the multiplicand, there are $3+3=6$ decimal places in the product.

(*c*) $.005^2 = .005 \times .005 = .000025$. Ans.

.0 0 5
.0 0 5
.0 0 0 0 2 5

Since there are 3 decimal places in the multiplicand and 3 in the multiplier, there are $3+3 = 6$ decimal places in the product.

(*d*) $.0063^2 = .0063 \times .0063 = .00003969$. Ans.

.0 0 6 3
.0 0 6 3
1 8 9
3 7 8
.0 0 0 0 3 9 6 9

Since there are 4 decimal places in the multiplicand and 4 in the multiplier, there are $4+4 = 8$ decimal places in the product.

(*e*) $10.06^2 = 10.06 \times 10.06 = 101.2036$. Ans.

1 0.0 6
1 0.0 6
6 0 3 6
1 0 0 6 0 0
1 0 1.2 0 3 6

Since there are 2 decimal places in the multiplicand and 2 in the multiplier, there are $2+2 = 4$ decimal places in the product.

(47) (*a*) $753^3 = 753 \times 753 \times 753 = 426{,}957{,}777$. Ans.

7 5 3
7 5 3
2 2 5 9
3 7 6 5
5 2 7 1
5 6 7 0 0 9
7 5 3
1 7 0 1 0 2 7
2 8 3 5 0 4 5
3 9 6 9 0 6 3
4 2 6 9 5 7 7 7 7

(*b*) $987.4^3 = 987.4 \times 987.4 \times 987.4 = 962{,}674{,}279.624$. Ans.

```
      987.4
      987.4
     -------
      39496
     69118
    78992
   88866
  ---------
  974958.76
      987.4
  ---------
  38998350 4
 6824711 32
7799670 08
877462884
---------
962674279.624
```

Since there is 1 decimal place in the multiplicand and 1 in the multiplier, there are $1+1=2$ decimal places in the first product.

Since there are 2 decimal places in the multiplicand and 1 in the multiplier, there are $2+1=3$ decimal places in the final product.

(*c*) $.005^3 = .005 \times .005 \times .005 = .000000125$. Ans.

Since there are 3 decimal places in the multiplicand and 3 in the multiplier, there are $3+3=6$ decimal places in the first product; but, as there are only 2 figures in the product, we prefix 4 ciphers to make the 6 decimal places.

```
    .005
    .005
   ------
  .000025
    .005
  -------
.000000125
```

Since there are 6 decimal places in the multiplicand and 3 in the multiplier, there are $6+3=9$ decimal places in the final product. In this case we prefix 6 ciphers to form the 9 decimal places.

(*d*) $.4044^3 = .4044 \times .4044 \times .4044 = .066135317184$. Ans.

```
      .4044
      .4044
     -------
      16176
     16176
    161760
   ---------
  .16353936
      .4044
  ---------
   65415744
  65415744
 654157440
-----------
.066135317184
```

Since there are 4 decimal places in the multiplicand and 4 in the multiplier, there are $4+4=8$ decimal places in the first product.

Since there are 8 decimal places in the second multiplicand and 4 in the multiplier, there are $8+4=12$ decimal places in the final product; but, as there are only 11 figures in the product, we prefix 1 cipher to make 12 decimal places.

(48) $2^5 = 2\times2\times2\times2\times2 = 32$. Ans.

(49) $3^4 = 3\times3\times3\times3 = 81$. Ans.

(50) (*a*) $67.85^2 = 67.85\times67.85 = 4{,}603.6225$. Ans.

$$\begin{array}{r} 67.85 \\ 67.85 \\ \hline 33925 \\ 54280 \\ 47495 \\ 40710 \\ \hline 4603.6225 \end{array}$$

Since there are 2 decimal places in the multiplier and 2 in the multiplicand, there are $2+2 = 4$ decimal places in the product.

(*b*) $967{,}845^2 = 967{,}845\times967{,}845 = 936{,}723{,}944{,}025$. Ans.

$$\begin{array}{r} 967845 \\ 967845 \\ \hline 4839225 \\ 38713800 \\ 77427600 \\ 677491500 \\ 5807070000 \\ 87106050000 \\ \hline 936723944025 \end{array}$$

(*c*) A fraction may be raised to any power by raising both numerator and denominator to the required term.

Thus, $\left(\frac{3}{8}\right)^2 = \frac{3}{8}\times\frac{3}{8} = \frac{3\times3}{8\times8} = \frac{9}{64}$. Ans.

(*d*) $\left(\frac{1}{4}\right)^2 = \frac{1}{4}\times\frac{1}{4} = \frac{1\times1}{4\times4} = \frac{1}{16}$. Ans.

(51) (*a*) $5^{10} = 5\times5\times5\times5\times5\times5\times5\times5\times5\times5 = 9{,}765{,}625$. Ans.

(*b*) $9^5 = 9\times9\times9\times9\times9 = 59{,}049$. Ans.

(*a*)

5
5
25
5
125
5
625
5
3125
5
15625
5
78125
5
390625
5
1953125
5
9765625

(*b*)

9
9
81
9
729
9
6561
9
59049

(52) (*a*) $1.2^4 = 1.2\times1.2\times1.2\times1.2 = 2.0736$. Ans.

1.2
1.2
24
12
1.44
1.2
288
144
1.728
1.2
3456
1728
2.0736

Since there is 1 decimal place in the multiplicand and 1 in the multiplier, we must point off $1+1 = 2$ decimal places in the first product.

Since there are 2 decimal places in the second multiplicand and 1 in the multiplier, we must point off $2+1 = 3$ decimal places in the second product.

Since there are 3 decimal places in the third multiplicand and 1 in the multiplier, we must point off $3+1 = 4$ decimal places in the final product.

(*b*) $11^6 = 11\times11\times11\times11\times11\times11 = 1{,}771{,}561$. Ans.

$$\begin{array}{r} 11 \\ 11 \\ \hline 121 \\ 11 \\ \hline 1331 \\ 11 \\ \hline 14641 \\ 11 \\ \hline 161051 \\ 11 \\ \hline 1771561 \end{array}$$

(*c*) $1^7 = 1\times1\times1\times1\times1\times1\times1 = 1$. Ans.

(*d*) $.01^4 = .01\times.01\times.01\times.01 = .00000001$. Ans.

Since there are 2 decimal places in the multiplicand and 2 in the multiplier, we must point off $2+2 = 4$ decimal places in the first product; but, as there is only 1 figure in the product, we prefix 3 ciphers to make the 4 necessary decimal places.

$$\begin{array}{r} .01 \\ .01 \\ \hline .0001 \\ .01 \\ \hline .000001 \\ .01 \\ \hline .00000001 \end{array}$$

Since there are 4 decimal places in the second multiplicand and 2 in the multiplier, we must point off $4+2 = 6$ decimal places in the second product. It is necessary to prefix 5 ciphers to make 6 decimal places.

Since there are 6 decimal places in the third multiplicand and 2 in the multiplier, we must point off $6+2 = 8$ decimal places in the product. It is necessary to prefix 7 ciphers to make 8 decimal places in the final product.

(*e*) $.1^5 = .1\times.1\times.1\times.1\times.1 = .00001$. Ans.

Since there is 1 decimal place in the multiplicand and 1 in the multiplier, we must point off $1+1 = 2$ decimal places

.1
.1
.01

in the first product. It is necessary to prefix 1 cipher to the product.

.1
.001

Since there are 2 decimal places in the second multiplicand and 1 in the multiplier, we must point off $2+1=3$ decimal places in the second product. It is necessary to prefix 2 ciphers to the second product.

.1
.0001

Since there are 3 decimal places in the third multiplicand and 1 in the multiplier, we must point off $3+1=4$ decimal places in the third product. It is necessary to prefix 3 ciphers to this product.

.1
.00001

Since there are 4 decimal places in the fourth multiplicand and 1 in the multiplier, we must point off $4+1=5$ decimal places in the final product. It is necessary to prefix 4 ciphers to this product.

(53) (*a*) $.0133^3 = .0133 \times .0133 \times .0133 = .000002352637$. Ans.

Since there are 4 decimal places in the multiplicand and 4 in the multiplier, we must point off $4+4=8$ decimal places in the product; but, as there are only 5 figures in the product, we prefix 3 ciphers to form the 8 necessary decimal places in the first product.

.0133
.0133
399
399
133
.00017689
.0133
53067
53067
17689
.000002352637

Since there are 8 decimal places in the multiplicand and 4 in the multiplier, we must point off $8+4=12$ decimal places in the product; but, as there are only 7 figures in the product, we prefix 5 ciphers to make the 12 necessary decimal places in the final product.

(*b*) $301.011^3 = 301.011 \times 301.011 \times 301.011 = 27,273,890.942264331$. Ans.

```
                  301.011
                  301.011
                  -------
                  301011
                 301011
                3010110
              9030330
              -------
              90607.622121
                  301.011
                  -------
              906076221 21
             90607622121
           906076221210
  27182286636 30
  ---------------
  27273890.942264331
```

Since there are 3 decimal places in the multiplicand and 3 in the multiplier, we must point off $3+3=6$ decimal places in the first product.

Since there are 6 decimal places in the multiplicand and 3 in the multiplier, we must point off $6+3=9$ decimal places in the final product.

(*c*) $(\frac{1}{8})^3 = \frac{1}{8}\times\frac{1}{8}\times\frac{1}{8} = \frac{1\times1\times1}{8\times8\times8} = \frac{1}{512}$. Ans.

(*d*) To find any power of a mixed number, first reduce it to an improper fraction, and then multiply the numerators together for the numerator of the answer, and multiply the denominators together for the denominator of the answer.

$(3\frac{3}{4})^3 = \frac{15}{4}\times\frac{15}{4}\times\frac{15}{4} = \frac{15\times15\times15}{4\times4\times4} = \frac{3,375}{64} = 52.734+$. Ans.

$$3\tfrac{3}{4} = \frac{3\times4+3}{4} = \frac{12+3}{4} = \tfrac{15}{4}.$$

```
  15          64)3375.000(52.734+.
  15             320
  --             ---
  75             175
 15              128
 ---             ---
 225             470
  15             448
 ----            ---
1125             220
 225             192
 ----            ---
3375             280
                 256
                 ---
                  24
```

Since *three* ciphers were annexed to the dividend, *three* decimal places must be pointed off in the quotient. It is easy to see that the next figure will be a 3; hence, write the sign +, as shown.

(54) Evolution is the reverse of involution. In involution we find the *power* of a number by multiplying the number by itself one or more times, while in evolution we find the *number* or *root* which was multiplied by itself one or more times to make the power.

(55) (*a*)

1	$\sqrt{3'4\ 8'6\ 7'8\ 4.4\ 0'1\ 0} = 1{,}867.29+$. Ans.
1	1
20	248
8	224
28	2467
8	2196
360	27184
6	26089
366	3734) 1095.000 (.293 or .29+
6	7468
3720	34820
7	33606
3727	1214 0
7	
3734	

Explanation.—Applying the short method described in Art. **91,** we extract the root by the regular method to four figures, since there are six figures in the answer, and $6 \div 2 + 1 = 4$. The last remainder is 1,095, and the last trial divisor (with the cipher omitted) is 3,734. Dividing 1,095 by 3,734, as shown, the quotient, is .293+, or .29+, using two figures. Annexing to the root, gives 1,867.29+. Ans.

$\sqrt{9'0\,0'0\,0'9\,9.4\,0'0\,9'0\,0} = 3{,}000.0165+.$ Ans.

(*b*)	(*a*)	3	(*b*)	9
		3		
	(*d*)	60	(*c*)	00009940009
		0		600001
		600		39400800
		0		36000156
		6000		3400644
		0		
		60000		
		0		
		600000		
		1		
		600001		
		1		
		60000020		
		6		
		60000026		

Explanation.—Beginning at the decimal point, we point the whole number into periods of two figures each, proceeding from right to left; also, point off the decimal into periods of two figures each, proceeding from left to right. The largest number whose square is contained in the first period, 9, is 3; hence, 3 is the first figure of the root. Place 3 at the left, as shown at (*a*), and multiply it by the first figure in the root, or 3. The result is 9. Write 9 under the first period, 9, as at (*b*), subtract, and there is no remainder. Bring down the next period, which is 00, as shown at (*c*). Add the root already found to the 3 at (*a*), obtaining 6, and annex a cipher to this 6, thus making it 60, which is the trial divisor, as shown at (*d*). Divide the dividend (*c*) by the trial divisor, and obtain 0 as the next figure in the root. Write 0 in the root, as shown, and also add it to the trial divisor, 60, and annex a cipher, thereby making the next trial divisor 600. Bring down the next period, 00, annex it to the dividend already obtained, and divide it by the trial divisor. 600 is contained in 0000, 0 times, so we place

another cipher in the root. Write 0 in the root, as shown, and also add it to the trial divisor, 600, and annex a cipher, thereby making the next trial divisor 6,000. Bring down the next period, 99. The trial divisor, 6,000, is contained in 000099, 0 times, so we place 0 as the next figure in the root, as shown, and also add it to the trial divisor, 6,000, and annex a cipher, thereby making the next trial divisor 60,000. Bring down the next period, 40, and annex it to the dividend already obtained to form the new dividend, 00009940, and divide it by the trial divisor, 60,000. 60,000 is contained in 00009940, 0 times, so we place another cipher in the root, as shown, and also add it to the trial divisor, 60,000, and annex one cipher, thereby making the next trial divisor 600,000. Bring down the next period, 09, and annex it to the dividend already obtained to form the new dividend, 0000994009, and divide it by the trial divisor, 600,000. 600,000 is contained in 0000994009, once, so we place 1 as the next figure in the root, and also add it to the trial divisor, 600,000, thereby making the complete divisor 600,001. Multiply the complete divisor, 600,001, by 1, the sixth figure in the root, and subtract the result obtained from the dividend. The remainder is 394,008, to which we annex the next period, 00, to form the next new dividend, or 39,400,800. Add the sixth figure of the root, or 1, to the divisor, 600,001, and annex a cipher, thus obtaining 6,000,020 as the next trial divisor. Dividing 39,400,800 by 6,000,020, we find 6 to be the next figure of the root. Adding this last figure, 6, to the trial divisor, we obtain 6,000,026 for our next complete divisor, which multiplied by the last figure of the root, or 6, gives 36,000,156, which write under 39,400,800 and subtract. Since there is a remainder, it is clearly evident that the given power is not a perfect square, so we place + after the root. Since the next figure is 5, the answer is 3,000.017—.

In this problem there are seven periods—four in the whole number and three in the decimal—hence, there will be seven figures in the root, four figures constituting the whole number and three figures the decimal of the root. Hence, $\sqrt{9,000,099.4009} = 3,000.017-$.

(*c*)

3	$\sqrt{.0\,0'1\,2'2\,5} = .035.$ Ans.
3	0 0
6 0	1 2
5	9
6 5	3 2 5
	3 2 5

Pointing off periods, we find that the first period is composed of ciphers; hence, the first figure of the root will be a cipher. No further explanation is necessary, since this problem is solved in Art. **83,** *Arithmetic*, § 2. Since there are three decimal periods in the power, there will be three decimal figures in the root.

(56) (*a*)

1	$\sqrt{1'0\,7'9\,5.2\,1} = 103.9.$ Ans.
1	1
2 0	0 7 9 5
0	6 0 9
2 0 0	1 8 6 2 1
3	1 8 6 2 1
2 0 3	
3	
2 0 6 0	
9	
2 0 6 9	

(*b*)

2	$\sqrt{7'3\,0'0\,8.0\,5} = 270.2+.$ Ans.
2	4
4 0	3 3 0
7	3 2 9
4 7	1 0 8 0 5
7	1 0 8 0 4
5 4 0 0	1
2	
5 4 0 2	

(*c*)

9	$\sqrt{90.00'00'00}$ = 9.487— Ans.	
9	81	
180	900	
4	736	
184	16400	
4	15104	
1880	1896) 1296.00 (.68+ or .7—	
8	11376	
1888	15840	
8	14168	
1896		

Having found the first three figures, we find the fourth by division, as shown.

(*d*) $\sqrt{.09}$ = .3. Ans.

(57)	(*a*)	
6	36	$\sqrt[3]{.327'680'000}$ = .6894+. Ans
6	72	216
12	10800	111680
6	1504	98432
180	12304	13248000
8	1568	12650769
188	1387200	1424163) 59723100 (.41+ or .4+
8	18441	56966652
196 .	14056441	2756580
8	18522	1424163
2040	14241663	
9		
2049		
9		
2058		
9		
2067		

Here we find the first three figures in the regular way, and the fourth figure by the short method. See Art. **103.**

Explanation.—(1) When extracting the cube root, we divide the power into periods of three figures each. Always

begin at the decimal point and proceed to the left in pointing off the whole number, and to the right in pointing off the decimal. In this power, $\sqrt[5]{.32768}$, a cipher must be annexed to 68 to complete the second decimal period. Cipher periods may now be annexed until the root has as many figures as desired.

(2) We find by trial that the largest number whose cube is contained in the first period, 327, is 6. Write 6 as the first figure of the root, also at the extreme left at the head of column (1). Multiply the 6 in column (1) by the first figure of the root, 6, and write the product, 36, at the head of column (2). Multiply the number in column (2) by the first figure of the root, 6, and write the product, 216, under the figures in the first period. Subtract and bring down the next period, 680, annexing it to the remainder, 111, thereby obtaining 111,680 for a new dividend. Add the first figure of the root, 6, to the number in column (1), obtaining 12, which we call the first correction; multiply the first correction, 12, by the first figure of the root, and we obtain 72 as the product, which added to 36 of column (2) gives 108. Annexing two ciphers to 108, we have 10,800 for the trial divisor. Dividing the dividend by the trial divisor, we see that it is contained about 8 times, so we write 8 as the second figure of the root. Add the first figure of the root to the first correction, and we obtain 18 as the second correction. To this annex one cipher, and add the second figure of the root, and we obtain 188. This multiplied by the second figure of the root, 8, equals 1,504, which added to the trial divisor, 10,800, forms the complete divisor, 12,304. Multiplying the complete divisor, 12,304, by 8, the second figure of the root, the result is 98,432. Write 98,432 under the dividend, 111,680; subtract, and there is a remainder of 13,248. To this remainder annex the next period, 000, thereby obtaining 13,248,000 for the next new dividend.

(3) Adding the second figure of the root, 8, to the number in column (1), 188, we have 196 for the first new correction. This multiplied by the second figure of the root, 8, gives 1,568. Adding this product to the last complete divisor, and

annexing two ciphers, gives 1,387,200 for the next trial divisor. Adding the second figure of the root, 8, to the first new correction, 196, we obtain 204 for the new second correction. Dividing the dividend by the trial divisor, 1,387,200, we see that it is contained about 9 times. Write 9 as the third figure of the root. Annex one cipher to the new second correction, and to this add the third figure of the root, 9, thereby obtaining 2,049. This multiplied by 9, the third figure of the root, equals 18,441, which added to the trial divisor, 1,387,200, forms the complete divisor, 1,405,641. Multiplying the complete divisor by the third figure of the root, 9, and subtracting, we have a remainder of 597,231. We then find the fourth figure by division, as shown.

(*b*)

4	16	$\sqrt[3]{74088} = 42.$ Ans.
4	32	64
8	4800	10088
4	244	10088
120	5044	
2		
122		

(*c*)

4	16	$\sqrt[3]{92416} = 45.212-.$ Ans.
4	32	64
8	4800	28416
4	625	27125
120	5425	1291000
5	650	1220408
125	607500	612912) 70592.000 (.115
5	2704	612912
130	610204	930080
5	2708	612912
1350	612912	3171680
2		3064560
1352		107120
2		
1354		

8-6

(*d*)

7	49	$\sqrt[3]{.373248} = .72$. Ans.
7	98	343
14	14700	30248
7	424	30248
210	15124	
2		
212		

(58)

1	1	$\sqrt[3]{2.000'000'000} = 1.259921+$. Ans.
1	2	1
2	300	1000
1	64	728
30	364	272000
2	68	225125
32	43200	46875000
2	1825	42491979
34	45025	4755243) 4383021.000 (.9217 or .922—
2	1850	42797187
360	4687500	10330230
5	33831	9510486
365	4721331	8197440
5	33912	4755243
370	4755243	34421970
5		
3750		
9		
3759		
9		
3768		

This example shows what a great saving of figures is effected by using the short method. The figures obtained by the division are 9,217, thus making the last figures of the answer 922, according to Art. **91.** This is not correct in this case, the true answer, to eight decimal places, being 1.25992104+; hence, the first three figures found by

division should be used in this case. The reason for the apparent failure of the method, in this case, to give the seventh figure of the root correctly, is because the fifth figure (the first obtained by division) is 9. Whenever the first figure obtained by division is 8 or 9, it is better to carry the root process one place further, before applying Art. **91,** if it is desired to obtain absolutely correct results.

(59) (*a*)

1	1	$\sqrt[3]{1'758.416'743} = 12.07.$ Ans.
1	2	1
2	300	758
1	64	728
30	364	30416743
2	68	30416743
32	4320000	
2	25249	
34	4345249	
2		
3600		
7		
3607		

(*b*)

1	1	$\sqrt[3]{1'191'016} = 106.$ Ans.
1	2	1
2	30000	191016
1	1836	191016
300	31836	
6		
306		

(*c*) $\sqrt[3]{\frac{4}{3\,2}} = \sqrt[3]{\frac{1}{8}} = \frac{\sqrt[3]{1}}{\sqrt[3]{8}} = \frac{1}{2}.$ Ans.

(*d*) $\sqrt[3]{\frac{27}{5\,1\,2}} = \frac{\sqrt[3]{27}}{\sqrt[3]{512}} = \frac{3}{8}.$ Ans.

(60)

1	1	
1	2	
2	300	
1	136	
30	436	
4	152	
34	58800	
4	1696	
38	60496	
4	1712	
420	6220800	
4	8644	
424	6229444	
4	8648	
428	6238092	
4		
4320		
2		
4322		
2		
4324		

$\sqrt[3]{3.000'000'000} = 1.442250-$. Ans.

1	
2000	
1744	
256000	
241984	
14016000	
12458888	
6238092)	1557112.000 (.2496 or .250–
	12476184
	3094936 0
	2495236 8
	599699 20
	561428 28
	38270 92

(61) (*a*)

1	$\sqrt{1'2\ 3.2\ 1} = 11.1$. Ans.
1	1
20	23
1	21
21	221
1	221
220	
1	
221	

(*b*)

1	$\sqrt{1'1\ 4.9\ 2'1\ 0} = 10.72+$. Ans.
1	1
200	1492
7	1449
207	4310
7	4284
2140	26
2	
2142	

(*c*)

7	$\sqrt{5\ 0'2\ 6'8\ 1} = 709$. Ans.
7	49
140	1268 1
0	1268 1
1400	
9	
1409	

(*d*)

2	$\sqrt{.0\ 0'0\ 4'1\ 2'0\ 9} = .0203$. Ans.
2	00
400	04
3	4
403	1209
	1209

(62) (*a*)

1	1	$\sqrt[3]{.006'500'000} = .18663—$. Ans.
1	2	1
2	300	5500
1	304	4832
30	604	668000
8	368	602856
38	97200	103788) 651144.00 (.627 or .63—
8	3276	622728
46	100476	287120
8	3312	207576
540	103788	79544
6		
546		
6		
552		

(*b*)

2	4	$\sqrt[3]{.021'000'000} = .2759—$. Ans.
2	8	8
4	1200	13000
2	469	11683
60	1669	1317000
7	518	1113875
67	218700	226875) 203125.0 (.89 or .9—
7	4075	1815000
74	222775	216250
7	4100	
810	226875	
5		
815		
5		
820		

(*c*)

2	4
2	8
4	12000000
2	18009
6000	12018009
3	
6003	

$\sqrt[3]{8'036'054'027} = 2{,}003.$ Ans.

8

036054027

36054027

(*d*)

1	1
1	2
2	300
1	216
30	516
6	
36	

$\sqrt[3]{.000'004'096} = .016.$ Ans.

000

004

1

3096

3096

(*e*)

2	4
2	8
4	1200
2	325
60	1525
5	350
65	187500
5	5299
70	192799
5	5348
750	198147
7	
757	
7	
764	

$\sqrt[3]{17.000'000} = 2.5713-.$ Ans.

8

9000

7625

13750000

13495937

198147) 2540700 (.128 or .13—

198147

559230

396294

162936

(63) (*a*) In this example, the index is 4, and equals 2×2. The root indicated is the fourth root, hence the square root must be extracted twice. Thus, $\sqrt[4]{\ } = \sqrt{\ }$ of the $\sqrt{\ }$ and $\sqrt[4]{6,561} = \sqrt{\sqrt{6,561}} = \sqrt{81} = 9$. Ans.

8	$\sqrt{6\ 5'6\ 1} = 81$	$\sqrt{81} = 9$. Ans.
8	6 4	
1 6 0	1 6 1	
1	1 6 1	
1 6 1		

(*b*) In this example the index is 6, and 6 equals 3×2 or 2×3. The root indicated is the sixth root; hence, extract both the square and the cube roots, it making no particular difference as to which root is extracted first. Thus, $\sqrt[6]{\ }$ $= \sqrt[3]{\ }$ of the $\sqrt{\ }$, or $\sqrt{\ }$ of the $\sqrt[3]{\ }$.

Hence, $\sqrt[6]{117,649} = \sqrt[3]{\sqrt{117,649}} = \sqrt[3]{343} = 7$. Ans.

3	$\sqrt{1\ 1'7\ 6'4\ 9} = 343$	$\sqrt[3]{3\ 4\ 3} = 7$. Ans.
3	9	
6 0	2 7 6	
4	2 5 6	
6 4	2 0 4 9	
4	2 0 4 9	
6 8 0		
3		
6 8 3		

(*c*) $\sqrt[6]{.000064} = \sqrt[3]{\sqrt{.000064}} = .2$. Ans.

$\sqrt{.000064} = .008$. $\sqrt[3]{.008} = .2$. Hence, $\sqrt[6]{.000064} = .2$. Ans.

(*d*) $\sqrt[3]{\frac{3}{8}} = ?$ $\frac{3}{8} = .375$, since 8)3.000
.375

7	49.	$\sqrt[3]{.375\,000\,000} = .72112+.$
7	98	343 Ans.
14	14700	32000
7	424	30248
210	15124	1752000
2	428	1557361
212	1555200	1559523) 194639.00 (.124 or .12+
2	2161	1559523
214	1557361	3868670
2	2162	3119046
2160	1559523	749624
1		
2161		
1		
2162		

Hence, $\sqrt[3]{\frac{3}{8}} = .72112+.$ Ans.

(64) (*a*) $\sqrt{\frac{1225}{5476}} = \frac{\sqrt{1,225}}{\sqrt{5,476}}.$

3	$\sqrt{12'25} = 35$
3	9
60	325
5	325
65	

7	$\sqrt{54'76} = 74$
7	49
140	576
4	576
144	

Hence, $\sqrt{\frac{1225}{5476}} = \frac{35}{74}.$ Ans.

(*b*)

```
 5   √.3 3'6 4 = .58.   Ans.
 5     2 5
100     8 6 4
  8     8 6 4
108
```

(*c*)

```
 3   √.1 0'0 0'0 0'0 0 = .31623—.   Ans.
 3     9
60     100
 1      61
61     3900
 1     3756
620    632 ) 144.00 (.227 or .23—
  6          1264
626           1760
  6           1264
632            496
```

(*d*) $25.0\frac{3}{4} = 25.075.$

```
 5   √2 5.0 7'5 0'0 0'0 0'0 0 = 5.00749+.   Ans.
 5     25
10000     075000
    7     70049
10007     495100
    7     400576
100140      9452400
    4       9013401
100144        438999
    4
1001480
    9
1001489
```

(*e*) $.000\frac{4}{9} = .0004444444+.$

```
 2   √.0 0'0 4'4 4'4 4'4 4 = .02108+.   Ans.
 2     00
40     04
 1      4
41     44
 1     41
4200    34444
   8    33664
4208      780
```

(65) (*a*) $\sqrt[4]{2} = \sqrt{\sqrt{2}}$.

$\sqrt{2.0\,0'0\,0'0\,0'0\,0} = 1.41421356+$

1	1
1	100
20	96
4	400
24	281
4	11900
280	11296
1	60400
281	56564
1	28284) 383 6.0000 (.13562 or .1356+
2820	28284
4	100760
2824	84852
4	159080
28280	141420
2	176600
28282	169704
2	6896
28284	

$\sqrt{1.41'42'13'56} = 1.1892+$. Ans.

1	1
1	41
20	21
1	2042
21	1824
1	21813
220	21321
8	49256
228	47564
8	1692
2360	
9	
2369	
9	
23780	
2	
23782	

It is required in this problem to extract the fourth root of 2 to 4 decimal places; hence, we must extract the square root twice, since $\sqrt[4]{\quad} = \sqrt{\quad}$ of the $\sqrt{\quad}$. In the first operation, we carry the root to 8 decimal places in order to carry the root in the second operation to 4 decimal places.

(*b*) $\sqrt[6]{6} = \sqrt[2]{\sqrt[3]{6}}$.

```
2          √6.00'00'00'00'00'00 = 2.4494897428+
2           4
40          200
 4          176
44           2400
 4           1936
480           46400
  4           44001
484            239900
  4            195936
4880            4396400
   9            3919104
4889    489896)477296.000000(.974280 or .97428+
   9            4409064
48980            3638960
    4            3429272
48984             2096880
    4             1959584
489880             1372960
     8              979792
489888              3931680
     8              3919168
489896                12512
```

It is required in this problem to find the sixth root of 6; hence, it is necessary to extract both the square and cube roots in succession, since the index, 6, equals 2×3 or 3×2. It makes no particular difference as to which root we extract first, but it will be more convenient to extract the square root first. The result has been carried to 10 decimal places; since the answer requires but 5 decimal places, the remaining decimals will not affect the cube root in the fifth decimal place, as the student can see for himself if he will continue the operation.

1	1
1	2
2	3 0 0
1	9 9
3 0	3 9 9
3	1 0 8
3 3	5 0 7 0 0
3	1 5 7 6
3 6	5 2 2 7 6
3	1 5 9 2
3 9 0	5 3 8 6 8 0 0
4	3 2 2 2 4
3 9 4	5 4 1 9 0 2 4
4	3 2 2 8 8
3 9 8	5 4 5 1 3 1 2
4	
4 0 2 0	
8	
4 0 2 8	
8	
4 0 3 6	

$\sqrt[3]{2.4\,4\,9'4\,8\,9'7\,4\,2'8\,0\,0} = 1.34801—.$ Ans.

1	
1 4 4 9	
1 1 9 7	
2 5 2 4 8 9	
2 0 9 1 0 4	
4 3 3 8 5 7 4 2	
4 3 3 5 2 1 9 2	
5451312) 3 3 5 5 0.0 0 0	(.006 or .01—
3 2 7 0 7 8 7 2	
8 4 2 1 2 8	

(66) (*a*)

1	
1	
2 0	
7	
2 7	
7	
3 4 0	
7	
3 4 7	
7	
3 5 4	

$\sqrt{3.1\,4'1\,6} = 1.7725—.$ Ans.

1	
2 1 4	
1 8 9	
2 5 1 6	
2 4 2 9	
354) 8 7.0 0	(.245+ or .25—
7 0 8	
1 6 2 0	
1 4 1 6	
2 0 4	

(*b*)

8	
8	
160	
8	
168	
8	
1760	
6	
1766	
6	
1772	

$\sqrt{.7\ 8'5\ 4'0\ 0} = .8862+$. Ans.

64	
1454	
1344	
11000	
10596	
1772)	404.0 (.22 or .2+
	3544
	496

(67) (*a*)

1	1
1	2
2	300
1	136
30	436
4	152
34	58800
4	2556
38	61356
4	2592
420	6394800
6	17536
426	6412336
6	17552
432	6429888
6	
4380	
4	
4384	
4	
4388	

$\sqrt[3]{3.1\ 4\ 1'6\ 0\ 0'0\ 0\ 0} = 1.4646-$. Ans.

1	
2141	
1744	
397600	
368136	
29464000	
25649344	
6429888)	3814656.0 (.59 or .6–
	32149440
	5997120

(b)

8	64	$\sqrt[3]{.523'600'000} = .80599+$ or
8	128	512 .8060—. Ans.
16	1920000	11600000
8	12025	9660125
2400	1932025	1944075) 19398750 (.99
5	12050	17496675
2405	1944075	1902075 0
5		17496675
2410		

(68) $11.7 : 13 :: 20 : x.$ The product of the means is equal to the product of the extremes.

$11.7x = 13 \times 20$

$11.7x = 260$

$x = \frac{260}{11.7}$) 260.000 (22.22+. Ans.

234

260

234

260

234

260

234

26

(69) (a) $20+7 : 10+8 :: 3 : x.$

$27 : 18 :: 3 : x.$

$27x = 18 \times 3$

$27x = 54$

$x = \frac{54}{27} = 2.$ Ans.

(b) $(12)^2 : (100)^2 :: 4 : x.$

$144 : 10{,}000 :: 4 : x$

$144x = 10{,}000 \times 4$

$144x = 40{,}000$

$$x = \frac{40{,}000}{144}$$

144) 4 0 0 0 0.0 (277.7+. Ans.

2 8 8

1 1 2 0

1 0 0 8

1 1 2 0

1 0 0 8

1 1 2 0

1 0 0 8

1 1 2

(70) (*a*) $\frac{4}{x} = \frac{7}{21}$ is equivalent to $4 : x :: 7 : 21$. The product of the means equals the product of the extremes. Hence,

$$7x = 4 \times 21$$
$$7x = 84$$
$$x = \frac{84}{7} \text{ or } 12. \quad \text{Ans.}$$

(*b*) In like manner,

$$\frac{x}{24} = \frac{8}{16} \text{ is equivalent to } x : 24 :: 8 : 16.$$
$$16x = 24 \times 8$$
$$16x = 192$$
$$x = \frac{192}{16} = 12. \quad \text{Ans.}$$

(*c*) $\frac{2}{10} = \frac{x}{100}$ is equivalent to $2 : 10 :: x : 100.$

$$10x = 2 \times 100$$
$$10x = 200$$
$$x = \frac{200}{10} = 20. \quad \text{Ans.}$$

(*d*) $\frac{15}{45} = \frac{60}{x}$ is equivalent to

$15 : 45 :: 60 : x.$

$15x = 45 \times 60$

$15x = 2{,}700$

$$x = \frac{2{,}700}{15} = 180. \quad \text{Ans.}$$

(*e*) $\frac{10}{150} = \frac{x}{600}$ is equivalent to

$10 : 150 :: x : 600.$

$150x = 10 \times 600$

$150x = 6{,}000$

$$x = \frac{6{,}000}{150} = 40. \quad \text{Ans.}$$

(71) $x : 5 :: 27 : 12.5.$

$$12.5 \overline{)\,1\,3\,5.0\,(\,10\tfrac{4}{5}.} \quad \text{Ans.}$$
$$\underline{1\,2\,5}$$
$$\underline{1\,0\,0}$$
$$\underline{1\,2\,5} = \tfrac{4}{5}.$$

(72) $45 : 60 :: x : 24.$

$$60x = 45 \times 24$$
$$60x = 1{,}080$$
$$x = \frac{1{,}080}{60} = 18. \quad \text{Ans.}$$

(73) $x : 35 :: 4 : 7.$

$$7x = 35 \times 4$$
$$7x = 140$$
$$x = \tfrac{140}{7} = 20. \quad \text{Ans.}$$

(74) $9 : x :: 6 : 24.$

$$6x = 9 \times 24$$
$$6x = 216$$
$$x = \tfrac{216}{6} = 36. \quad \text{Ans.}$$

(75)

$\sqrt[3]{1{,}000} : \sqrt[3]{1{,}331} :: 27 : x.$

$10 : 11 :: 27 : x.$

$$10x = 297$$
$$x = \tfrac{297}{10} = 29.7. \quad \text{Ans.}$$

$$\begin{array}{r} 1 \\ \underline{1} \\ 2 \\ \underline{1} \\ 3\,0 \\ \underline{1} \\ 3\,1 \end{array}$$

$\sqrt[3]{1{,}000} = 10.$

$\sqrt[3]{1{,}331} = 11.$

$$\begin{array}{r} 1 \\ \underline{2} \\ 3\,0\,0 \\ \underline{3\,1} \\ 3\,3\,1 \end{array}$$

$$1'3\,3\,1\,(11$$
$$\underline{1}$$
$$3\,3\,1$$
$$\underline{3\,3\,1}$$

(76) $64 : 81 :: 21^3 : x^3.$

Extracting the square root of each term of any proportion does not change its value, so we find that $\sqrt{64} : \sqrt{81} :: \sqrt{21^3} : \sqrt{x^3}$ is the same as

$$8 : 9 :: 21 : x.$$
$$8x = 189$$
$$x = 23.625. \quad \text{Ans.}$$

(77) $7 + 8 : 7 :: 30 : x$ is equivalent to

$$15 : 7 :: 30 : x.$$
$$15x = 7 \times 30$$
$$15x = 210$$
$$x = \tfrac{210}{15} = 14. \quad \text{Ans.}$$

(78) 2 ft. 5 in. = 29 in.; 2 ft. 7 in. = 31 in. Stating as a direct proportion, $29:31 = 2{,}480:x$. Now, it is easy to see that x will be greater than 2,480. But x should be less than 2,480, since when a man lengthens his steps, the number of steps required for the same distance is less; hence, the proportion is an inverse one, and

$$29:31 = x:2{,}480,$$
or $$31x = 71{,}920;$$
whence, $$x = 71{,}920 \div 31 = 2{,}320 \text{ steps.}$$ Ans.

(79) This is evidently a direct proportion. 1 hr. 36 min. = 96 min.; 15 hr. = 900 min. Hence,

$$96:900 = 12:x,$$
or $$96x = 10{,}800;$$
whence, $$x = 10{,}800 \div 96 = 112.5 \text{ mi.}$$ Ans.

(80) This is also a direct proportion; hence,

$$27.63:29.4 = .76:x,$$
or $$27.63x = 29.4 \times .76 = 22.344;$$
whence, $$x = 22.344 \div 27.63 = .808+ \text{ lb.}$$ Ans.

(81) 2 gal. 3 qt. 1 pt. = 23 pt.; 5 gal. 3 qt. = 46 pt.

Hence, $$23:46 = 5:x,$$
or $$23x = 46 \times 5 = 230;$$
whence, $$x = 230 \div 23 = 10 \text{ days.}$$ Ans.

(82) Stating as a direct proportion, and squaring the distances, as directed by the statement of the example, $6^2:12^2 = 24:x$. Inverting the second couplet, since this is an inverse proportion,

$$6^2:12^2 = x:24.$$

Dividing both terms of the first couplet (see Art. **129**) by 6,

$$1^2:2^2 = x:24; \text{ or, } 1:4 = x:24;$$
whence, $4x = 24$, or $x = 6$ degrees. Ans.

(83) Taking the dimensions as the causes,

$$\begin{array}{|c|c|} \hline 12 & 15 \\ 4 & 5 \\ 2 & 7 \\ 3 & 6 \\ \hline \end{array} = 12 \Big| \; x, \text{ whence, } 2x = 75, \text{ or } x = \$37.50. \text{ Ans.}$$

(84) 2 hr. = 120 min.; 14 hr. 28 min. = 868 min. Hence,

$$120 : 868 = 100 : x,$$

or $$120x = 86{,}800;$$

whence, $$x = 723\tfrac{1}{3} \text{ gal. Ans.}$$

(85) Taking the dimensions as the causes,

$$\begin{array}{|c|c|} \hline 14 & 2 \\ 28 & 20 \\ 2 & \\ 12 & 17 \\ 10 & 6 \\ \hline \end{array} = 510 \Big| \; x, \text{ whence, } 14x = 17 \times 255 = 4{,}335,$$

or $x = 309\frac{9}{14}$ bbl. Ans.

(86) 8 hr. 40 min. = 520 min. Hence,

$$444 : 1{,}060 = 520 : x,$$

$$\text{or } x = \frac{1{,}060 \times \overset{130}{\cancel{520}}}{\underset{111}{\cancel{444}}} = \frac{137{,}800}{111} = 1{,}241.44+ \text{ min.} = 20 \text{ hr.}$$

41.44+ min. Ans.

(87) 1 min. = 60 sec. Hence,

$$5\tfrac{1}{2} : 60 = 6{,}160 : x,$$

or $$x = \frac{60 \times 6{,}160}{5.5} = 67{,}200 \text{ ft. Ans.}$$

(88) Writing the statement as a direct proportion, $8 : 10 = 5 : x$, it is easy to see that x will be greater than 5; but, it should be smaller, since by working longer hours, fewer men will be required to do the same work. Hence, the proportion is inverse. Inverting the second couplet,

$$8 : 10 = x : 5,$$

or $$x = \frac{\overset{4}{\cancel{8}} \times 5}{\underset{2}{\cancel{10}}} = 4 \text{ men. Ans.}$$

(89) Taking the times as the causes,

$$\begin{array}{c|c|c} 20 & 25 & 14 \\ & 5 & 70 \\ & = 540 & 630; \text{ whence, } 3x = 2\times 14 = 28, \text{ or } x \\ 10 & x \quad 27 & = 9\tfrac{1}{3} \text{ hr. Ans.} \\ 2 & 3 & \end{array}$$

(90) Taking the horsepowers as the effects, we have for the known causes in example 4, Art. **168,** 14^2, 500, and 48, and for the known effect 112 horsepower. Hence,

$$\begin{array}{c|c|c} 14^2 & 30^2 & 9 \\ 500 & 660 = 112 & x, \text{ or } 500 \quad 660 = 112 \quad x; \\ 48 & 42 & 6 \quad 3 \quad 8 \\ & & 48 \quad 42 \end{array}$$

whence, $x = 9\times 22\times 3 = 594$ horsepower. Ans.

(91) First find the volume of the cylinder in cubic inches, as in the example, Art. **164.** The volume, multiplied by the weight of 1 cubic inch (.261 lb.) will evidently be the weight of the cylinder. Hence,

$$\begin{array}{c|c|c} 10^2 & 12^2 & 100 \quad 144 \\ 20 & 60 = 1{,}570.8 & x, \text{ or } \quad 3 \quad = 1{,}570.8 \quad x; \\ & & 20 \quad 60 \end{array}$$

whence, $x = \dfrac{144\times 3\times 1{,}570.8}{100} = 1{,}771.11$ lb. Ans.

(92) Referring to the example in Art. **167,**

$$\begin{array}{c|c|c} & & 5 \quad 4 \\ & & 15 \quad 40 \\ 15 & 40 & 100 \\ 20^2 & 18^2 = 187 & x, \text{ or } 400 \quad 324 = 187 \quad x; \\ 10 & 12 & 4 \\ & & 10 \quad 12 \end{array}$$

whence, $x = \dfrac{324\times 4\times 187}{500} = 484.7$ lb. Ans.

FORMULAS.

(QUESTIONS 1-9. SEC. 3.)

(1) Substituting for D, x, B, and i their values,

$$C = \frac{D-x}{B+i} = \frac{120-12}{10+3.5} = \frac{108}{13.5} = 8. \quad \text{Ans.}$$

A line between two numbers signifies that the one above the line, or numerator, is to be divided by the one below the line, or denominator.

(2) Substituting for A, h, D, and x their values,

$$Q = \frac{A\,h+D}{2x+6}+D = \frac{(5\times 200)+120}{(2\times 12)+6} = \frac{1{,}000+120}{24+6}$$

$$= \frac{1{,}120}{30} = 37\tfrac{1}{3}.$$

$$37\tfrac{1}{3}+D = 37\tfrac{1}{3}+120 = 157\tfrac{1}{3}. \quad \text{Ans.}$$

When there is no sign between the letters, multiplication is understood.

(3) Substituting for B, h, A, x, and i their values,

$$r = \frac{3.246\times B\times h}{\frac{A\,x+h}{A\,i-B}} = \frac{3.246\times 10\times 200}{\frac{(5\times 12)+200}{(5\times 3.5)-10}} = \frac{6{,}492}{\frac{260}{7.5}}$$

$$= 6{,}492\div\frac{260}{7.5} = 6{,}492\times\frac{7.5}{260} = 187.269+. \quad \text{Ans.}$$

(4) Substituting for A, D, i, and B their values,

$$v = \sqrt{\frac{A\,D}{i\,B+1.5}} = \sqrt{\frac{5\times 120}{(3.5\times 10)+1.5}} = \sqrt{\frac{600}{36.5}}$$

$$= \sqrt{16.4383} = 4.05+. \quad \text{Ans.}$$

The square root sign extends over both numerator and denominator, thus indicating that the square root of the entire fraction is to be extracted.

(5) Substituting for B, x, h, and A their values,

$$u = \sqrt[3]{\frac{B\,x}{.00018\,h\,(A^2-x)}} = \sqrt[3]{\frac{10\times 12}{.00018\times 200\times(5^2-12)}}$$

$$= \sqrt[3]{\frac{120}{.036\times(25-12)}} = \sqrt[3]{\frac{120}{.036\times 13}} = \sqrt[3]{\frac{120}{.468}}$$

$$= \sqrt[3]{256.41+} = 6.35+. \quad \text{Ans.}$$

(6) Substituting for h, D, and A their values,

$$f = \frac{10\,(h-D)^2}{\sqrt[3]{D+A}} = \frac{10\,(200-120)^2}{\sqrt[3]{120+5}} = \frac{10\times 80^2}{\sqrt[3]{125}} = \frac{64{,}000}{5}$$

$$= 12{,}800. \quad \text{Ans.}$$

(7) Substituting for B, A, and D their values,

$$g = \frac{(B-A)^2-\sqrt[3]{D+A}}{A^2-(1+D)} = \frac{(10-5)^2-\sqrt[3]{120+5}}{5^2-(1+120)} = \frac{5^2-\sqrt[3]{125}}{125-121}$$

$$= \frac{25-5}{4} = \frac{20}{4} = 5. \quad \text{Ans.}$$

(8) Substituting for A, B, and h their values,

$$k = \sqrt{\frac{A\,B^2}{\sqrt[3]{A\,h}}} = \sqrt{\frac{5\times 10^2}{\sqrt[3]{5\times 200}}} = \sqrt{\frac{5\times 100}{\sqrt[3]{1{,}000}}} = \sqrt{\frac{500}{10}}$$

$$= \sqrt{50} = 7.071+. \quad \text{Ans.}$$

(9) Substituting for A, h, D, x, and B their values,

$$T = \sqrt{\frac{A^2\left[490+\frac{(h\,x)^2}{D^2}\right]}{h+\frac{x}{D}\,(A^2-B)^2}} = \sqrt{\frac{5^2\left[490+\frac{(200\times 12)^2}{120^2}\right]}{200+\frac{12}{120}\,(5^2-10)^2}}$$

$$= \sqrt{\frac{25\,(490+400)}{200+(\frac{1}{10}\times 225)}} = \sqrt{\frac{25\times 890}{200+22.5}} = \sqrt{\frac{22{,}250}{222.5}}$$

$$= \sqrt{100} = 10. \quad \text{Ans.}$$

GEOMETRY AND MENSURATION.

(QUESTIONS 1–42. SEC. 4.)

(1) (*a*) The angles of the triangles being equal, the bevel angle is 60°. Ans.

(*b*) $CE = 2.5$ ft. $OG = \sqrt{(OC)^2-(GC)^2} = \sqrt{(2.5)^2-(1.25)^2}$ $= \sqrt{4.69} = 2.16$ ft. $DF : CE = OH : OG$; or, $DF = \frac{CE \times OH}{OG} = \frac{2.5 \times 2.5}{2.16} = 2.89$ ft. $= 2$ ft. 10.7 in. Ans.

(*c*) Since the width, as IH, is 8 in. $= .67$ ft., $OI = 2.5 - .67 = 1.83$ ft. Then, $AB : DF = OI : OH$; or, $AB = \frac{DF \times OI}{OH} = \frac{2.89 \times 1.83}{2.5} = 2.12$ ft. $= 2$ ft. $1\frac{1}{2}$ in., nearly. Ans.

(2) By drawing a diagram, it will be seen that $\frac{1}{2}$ of 12 ft. $= 6$ ft. is the longer side of a right triangle, whose hypotenuse is $\frac{1}{2}$ of 13 ft. $= 6.5$ ft. Then the shorter side of the triangle $= \sqrt{(6.5)^2-6^2} = 2.5$ ft., which is one-half a side of the octagon. Hence, the entire perimeter of the octagon is $2.5 \times 2 \times 8 = 40$ ft. Ans.

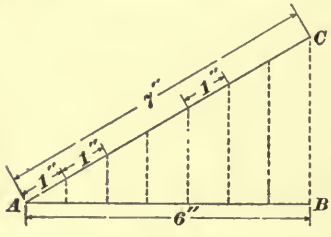
(3) (*a*) $CD = OD - OC$. $OC = \sqrt{(AO)^2-(AC)^2}$ $= \sqrt{3^2-(2.25)^2} = 1.98$ ft. $OD = 3$ ft.; hence, $CD = 3 - 1.98 = 1.02$ ft. $= 1$ ft. $\frac{1}{4}$ in. Ans.

(*b*) $OC = OD - CD$; $AB = 2AC = 2\sqrt{(OA)^2-(OC)^2}$. Ans.

(4) $FB = \sqrt{24^2+18^2} = 30$ ft. $FD = \sqrt{30^2+14^2} = 33.11$ ft.; $EA = \sqrt{24^2+30^2} = 38.42$ ft.; $EC = \sqrt{12^2+14^2} = 18.44$ ft. Ans.

(5) Because the angle included between lines drawn from the center to the ends of a side includes $\frac{1}{6}$ of 360°, or 60°, and the other angles are $\frac{1}{2}(180°-60°) = 60°$; hence, the triangle is equiangular, and, therefore, equilateral, making one side of a regular hexagon equal to the radius of the circumscribing circle.

(6) The sides of the roof being equal sides of a right triangle, of which the span is the hypotenuse, each side is equal to $\sqrt{\frac{24^2}{2}} = \sqrt{288}$. Ans.



(7) Let AB (see figure) be the line 6 in. long. Draw AC = 7 in., and connect C and B. Draw parallels to CB through each inch mark on AC. These lines will divide AB as required.

(8) Since the angles DCA and CDB are each 45° and AE is perpendicular to CD, $CE = FD = AE = 10$ ft.; therefore, $AB = 36-(10\times2)$ = 16 ft. Ans.

(9) (*a*) Since the side slopes are half pitch, $FG = FH = \frac{1}{2}AB$, or 11 ft. Then, $GH = \sqrt{(FH)^2+(FG)^2} = \sqrt{11^2+11^2} = 15.56$ ft., or 15 ft. $6\frac{3}{4}$ in., the length of a common rafter.

(*b*) $BC = \sqrt{(BD)^2+(DC)^2}$, but $BD^2 = ED^2+BE^2 = 11^2+11^2$; $DC = 11$ ft.; therefore, $BC = \sqrt{11^2+11^2+11^2} = 19.05$ ft., or 19 ft. $\frac{5}{8}$ in. Ans.

(10) Distance $AE = EB = 210\div2 = 105$ ft. Then, $AC:EG = AB:EB$; or, $AC = \frac{EG\times AB}{EB} = \frac{90\times210}{105} = 180$ ft. Ans.

As $BD : FE = AB : AE$, $BD = \frac{FE \times AB}{AE} = \frac{86 \times 210}{105}$ = 172 ft. Ans.

(11) (*a*) $90° - 37° = 53°$. Ans.
(*b*) $180° - 37° = 143°$. Ans.

(12) The angle included between two radii drawn to the ends of a side of a regular nonagon, is, since the sides are equal, $360° \div 9 = 40°$. In any triangle, the sum of the angles is 180°; hence, the sum of the other two angles is $180° - 40° = 140°$; the angles being equal, each is $\frac{1}{2}$ of 140°, or 70°. As a radius drawn to the angle of a regular polygon bisects it, then the angle between two sides of the polygon is twice 70°, or 140°. Ans.

(13) Since a perpendicular to a diameter from a point on the circumference is a mean proportional between the two parts of the diameter, $FB^2 = AB \times BC$; or, $FB = \sqrt{AB \times BC} = \sqrt{4 \times 12} = 6.93$ ft.; or, thus, $OB = AO - AB = 4$ ft., and $FB = \sqrt{(OF)^2 - (OB)^2} = \sqrt{8^2 - 4^2} = 6.93$ ft. Hence, $DF = 11$ ft. $- 6.93$ ft. $= 4.07$ ft. Ans.

(14) One-half of 32 ft. is 16 ft., which is one side of a right triangle, and the required length, 12 ft., is the other side. The hypotenuse is $\sqrt{12^2 + 16^2} = 20$ ft. Hold the 20-foot mark on the tape at one end of the line 32 ft. long, the 36-foot mark at the middle point, and bring the 48-foot mark and the end of the tape to an intersection, when drawn taut. Connect this point and the middle point of the 32-foot line. (Any method involving the proper application of right triangles is correct.)

(15) The difference is merely the weight taken off by the rounded corners. At each corner the deduction in area is equal to $\frac{1}{4}$ the difference between a circle of 2-inch radius and a square of twice the radius, and for 4 corners it is 4 times this, or the entire difference. The area of a 4-inch square is 16 sq. in.; that of a circle of 2-inch radius is 12.57 sq. in.; the difference is $16 - 12.57 = 3.43$ sq. in. The

thickness being $1\frac{1}{4}$ in., the volume is $3.43 \times 1.25 = 4.29$ cu. in., and at .26 lb. per cu. in., the weight is 1.11 lb. Ans.

(16) 30 in. = 2.5 ft.; 18 in. = 1.5 ft. Area of lower base is $2.5 \times 2.5 = 6.25$ sq. ft.; of upper base is $1.5 \times 1.5 = 2.25$ sq. ft. By the formula for frustum of pyramid, the volume is

$$\frac{h}{3}(A + a + \sqrt{A \times a}) = \tfrac{8}{3}(6.25 + 2.25 + \sqrt{6.25 \times 2.25})$$

$= 2(8.50 + 3.75) = 24.5$ cu. ft. Or, in the prismoidal formula, the mean area being $\left(\frac{2.5 + 1.5}{2}\right)^2 = 4$ sq. ft., the volume is $\frac{h}{6}(A + a + 4M) = \frac{8}{6}(6.25 + 2.25 + 16) = 24.5$ cu. ft. As there are $22\frac{1}{2}$ brick to a cu. ft., each pier will contain $22.5 \times 24.5 = 551.25$ brick; 18 piers will therefore require $551.25 \times 18 = 9{,}922.5$ brick. Ans.

(17) The area of the ring is the difference between a 7-inch and a 6-inch circle, or $\frac{\pi}{4}(7^2 - 6^2) = 10.21$ sq. in.; that of the flanges $= \frac{3}{4}$ in. $\times 1$ in. $\times 4 = 3$ sq. in., making a total of 13.21 sq. in. Since a strip 1 foot long and 1 inch square weighs 3.33 lb., 1 foot of the column will weigh $13.21 \times 3.33 = 43.99$ lb. Increasing this by 2 per cent., the weight per foot is $43.99 \times 1.02 = 44.87$ lb. Ans.

(18) Reducing feet and inches to feet and decimals:

1 piece,	$4.5 \times 3.75 \times 1.25$	=	2 1.0 9 cu. ft.
1 piece,	$4.25 \times 3.5 \times 1.25$	=	1 8.5 9 cu. ft.
1 piece,	$5.25 \times 4.0 \times 1.25$	=	2 6.2 5 cu. ft.
3 pieces,	$3.25 \times 3.5 \times 1.25$	=	4 2.6 6 cu. ft.
2 pieces,	$5.5 \times 4.0 \times 1.25$	=	5 5.0 0 cu. ft.
1 piece,	$5.0 \times 3.75 \times 1.25$	=	2 3.4 4 cu. ft.
1 piece,	$4.75 \times 4.5 \times 1.25$	=	2 6.7 2 cu. ft.
2 pieces,	$4.5 \times 4.25 \times 1.25$	=	4 7.8 2 cu. ft.
Total,			2 6 1.5 7 cu. ft.

261.57 cu. ft., at 140 lb. per cu. ft., weigh 36,619.8 lb. = 366.2 hundred pounds. At 21 cents per hundred, the freight charges are $366.2 \times \$.21 = \$76.90$. Ans.

(19) (*a*) MD, DL, KB, BI, AH, and AG are all equal, and their true length is the hypotenuse, as $M'D'$, of a right triangle, the sides of which, as $M'D''$ and $D'D''$, are each 10 ft.; hence, $M'D'$, etc. $= \sqrt{10^2+10^2} = 14.14$ ft.

Area, $ABIH = H'A' \times HI = 14.14 \times 36 =$ 5 0 9.0 4 sq.ft.

Area, $GFCA = \frac{1}{2}(GF+AC) \times A'G'$

$= \frac{12+22}{2} \times 14.14 =$ 2 4 0.3 8 sq.ft.

Area, $EKBC = \frac{1}{2}(EK+CB) \times K'B'$

$= \frac{4+14}{2} \times 14.14 =$ 1 2 7.2 6 sq.ft.

Area, $MDCF$ and $DLEC$

$= 2\left[\frac{1}{2}(MF+DC) \times M'D'\right]$

$= 2\left(\frac{10+20}{2} \times 14.14\right) =$ 4 2 4.2 0 sq.ft.

Total, 1,3 0 0.8 8 sq.ft.

Ans.

(*b*) The length of the valley FC is the hypotenuse of a right triangle *on the roof surface* of which the real length of FN (which is $M'D'$) and NC ($= AG = 10$ ft.) are the sides; hence, $FC = \sqrt{(M'D')^2+(NC)^2} = \sqrt{(14.14)^2+10^2}$ $= \sqrt{300} = 17.32$ ft. $= 17$ ft. $3\frac{7}{8}$ in. Ans.

(20) The quantity discharged is the volume of a cylinder whose length is $5 \times 60 = 300$ ft.; the area of a 4-inch circle is 12.57 sq. in. $= .09$ sq. ft.; hence, the volume is $.09 \times 300$ $= 27$ cu. ft., which, reduced to gallons, gives 27×7.5 $= 202.5$ gal. Ans.

(21) The volume of the metal will be $3 \div .41 = 7.32$ cu. in. The area of a $1\frac{3}{4}$-inch circle is 2.4 sq. in.; that of a $1\frac{1}{2}$-inch circle $= 1.77$ sq. in.; the area of the ring is, therefore, $2.40 - 1.77 = 0.63$ sq. in.; hence, the length required is $7.32 \div 0.63 = 11.6$ in. Ans.

(22) (*a*) In the formula

$$r = \frac{c^2+4h^2}{8h},$$

$c = 6$ ft. 8 in. $= 80$ in., and $h = 8$ in. Substituting,

$$r = \frac{80^2+4\times8^2}{8\times8} = 104 \text{ in.} = 8 \text{ ft. } 8 \text{ in.} \quad \text{Ans.}$$

(*b*) In the formula

$$l = \frac{4\sqrt{c^2 + 4h^2} - c}{3},$$

$c = 80$ in., $h = 8$ in. Substituting,

$$l = \frac{4\sqrt{6{,}400 + 256} - 80}{3} = \frac{4 \times 81.58 - 80}{3} = 82.11 \text{ in.},$$

the length of arc; as there are 13 stones, the length of each is $82.11 \div 13 = 6.32$ in. Ans.

(23) Since AC is a mean proportional to the height CD and the other part of the diameter, which we will denote by S; then,

$$S \times CD = \overline{AC}^2; \text{ or, } S = \frac{\overline{AC}^2}{CD} = \frac{40^2}{8} = 200 \text{ in.},$$

The diameter is the sum of the two parts, or $S + CD = 200 + 8 = 208$ in. The radius is, therefore, $208 \div 2 = 104$ in., or 8 ft. 8 in. Ans.

(24) The area required is the difference in areas of a 12-inch circle, and the segment ADB. The area of the circle is $.7854 \times 12^2 = 113.10$ sq. in. The length $AC = \sqrt{(AO)^2 - (OC)^2} = \sqrt{6^2 - 3^2} = 5.2$ in., and $AB = 5.2 \times 2 = 10.4$ in. The length of the arc ADB is found by formula

$$l = \frac{4\sqrt{c^2 + 4h^2} - c}{3}.$$

$c = AB = 10.4$ in.; $h = CD = 3$ in.; hence,

$$l = \frac{4\sqrt{108.16 + 36} - 10.4}{3} = \frac{4 \times 12.0 - 10.4}{3} = 12.53 \text{ in.}$$

The area of sector

$$OADB = \frac{lr}{2} = \frac{12.53 \times 6}{2} = 37.59 \text{ sq. in.};$$

the area of triangle is $\frac{10.4 \times 3}{2} = 15.60$ sq. in., whence, area of segment $ADB = 37.59 - 15.60 = 21.99$ sq. in. Deducting 21.99 sq. in. from 113.10 sq. in., the area of the circle, the remainder is 91.11 sq. in. Ans.

(25)

3 pieces,	3″×10″×16′ 0″	= 1 2 0 ft. B. M.
8 pieces,	3″×10″×12′ 6″	= 2 5 0 ft. B. M.
6 pieces,	3″× 8″×14′ 6″	= 1 7 4 ft. B. M.
10 pieces,	3″× 8″× 9′ 0″	= 1 8 0 ft. B. M
4 pieces,	3″× 6″×10′ 6″	= 6 3 ft. B. M.
20 pieces,	3″× 6″× 7′ 0″	= 2 1 0 ft. B. M.
8 pieces,	4″× 6″×18′ 0″	= 2 8 8 ft. B. M.
4 pieces,	4″× 6″× 6′ 6″	= 5 2 ft. B. M
52 pieces,	2″× 4″×18′ 0″	= 6 2 4 ft. B. M.
15 pieces,	2″× 4″× 9′ 6″	= 9 5 ft. B. M.
600 ft. B. M.	1″×6″ flooring	= 6 0 0 ft. B. M.
200 ft. B. M.	2-inch plank	= 2 0 0 ft. B. M.
Total,		2,8 5 6 ft. B. M. Ans.

(26) Perimeter of the house is $12+10+12+14+24+24 = 96$ ft. The height of it being 22 ft., the area is $96\times22 = 2{,}112$ sq. ft. Adding for gables, one $\frac{14\times7}{2} = 49$ sq. ft., and one $\frac{12\times7}{2} = 42$ sq. ft., the total area is $2{,}112+49+42 = 2{,}203$ sq. ft. Deduct 10 windows, 3 ft. $\times$ 7 ft. = 210 sq. ft.; 10 windows, 3 ft. $\times$ 6 ft. 0 in. = 180 sq. ft.; and 4 doors, 3 ft. 6 in. $\times$ 8 ft. = 112 sq. ft. The total deduction is $210+180+112 = 502$ sq. ft.; therefore, the net surface is $2{,}203-502 = 1{,}701$ sq. ft., requiring, at 7 bricks per sq. ft., $1{,}701\times7 = 11{,}907$ bricks. Ans.

(27) The major axis of the glass is 8 ft. $-$ (4 in. $\times$ 2) = 7 ft. 4 in. = 7.33 ft.; the minor axis is 5 ft. $-$ (4 in. $\times$ 2) = 4 ft. 4 in. = 4.33 ft. Then the area is, by the formula $A = .7854dD = .7854\times4.33\times7.33 = 24.93$ sq. ft. Ans.

(28) The area of the semielliptical arch is one-half of $.7854\,dD$; $d = 4\times2 = 8$ ft.; $D = 10$ ft.; hence, the area is $\frac{1}{2}(.7854\times8\times10) = 31.42$ sq. ft. Adding this to the rectangular area, 6 ft. $\times$ 10 ft. = 60 sq. ft., the total area is 91.42 sq. ft., which exceeds the required area, 88 sq. ft., by 3.42 sq. ft. **Ans.**

(29) Find the area of cross-section, beginning at the bottom:

	7.5 ft. × 1 ft. =	7.5 0 sq. ft.
	6.5 ft. × 1 ft. =	6.5 0 sq. ft.
Triangular portion,	$\frac{1.25 \text{ ft.} \times 15 \text{ ft.}}{2}$ =	9.3 7 sq. ft.
Center part,	2 ft. × 15 ft. =	3 0.0 0 sq. ft.
Under "steps"—		
1 strip,	3.7 5	
1 strip (3.75 × 2),	7.5 0	.75 ft. × 22.5 = 1 6.8 7 sq. ft.
1 strip (3.75 × 3),	1 1.2 5	
Total,	2 2.5 0	
Coping,	2.5 ft. × .5 ft. =	1.2 5 sq. ft.
Total area of cross-sections,		7 1.4 9 sq. ft.

The length being 16 ft., the contents are 71.49 × 16 = 1,143.84 cu. ft. = 1,143.84 ÷ 27 = 42.36 cu. yd. Ans.

(30) In the formula

$$l = \frac{4\sqrt{c^2 + 4h^2} - c}{3},$$

$c = 3\frac{1}{2}$ ft. = 42 in.; $h = 3$ in.; substituting,

$$l = \frac{4\sqrt{42^2 + 3^2 \times 4} - 42}{3} = \frac{4\sqrt{1{,}800} - 42}{3} = 42.57 \text{ in.} = 42\tfrac{9}{16} \text{ in.},$$

or 3 ft. $6\frac{9}{16}$ in. Ans.

(31) $AF + 2$ ft. = 26 ft.; $AB + 2$ ft. = 20 ft.

$AF \times AB$ = 5 2 0 sq. ft.

CD = 12 ft.; $DE + 2$ ft. = 16 ft.

$CD \times DE$ = 1 9 2 sq. ft.

Total, 7 1 2 sq. ft.

712 × 9 = 6,408 cu. ft., and 6,408 ÷ 27 = 237.33 cu. yd. Ans.

(32) (*a*) Since the height of the gables is $\frac{5}{8}$ the width, 24 ft., it is $\frac{5}{8} \times 24$ = 15 ft. The length of the roof slope is the hypotenuse of a right triangle, the sides of which are 15 ft. and 12 ft. (one-half the width); it is, therefore, $\sqrt{15^2 + 12^2}$ = 19.21 ft. Adding 15 in. = 1.25 ft., the total width of each half of the roof is 19.21 + 1.25 = 20.46 ft. The length

is 32 ft. + (15 in. × 2) = 34.5 ft. The area of each half is 20.46 × 34.5 = 705.87 sq. ft.; the area of the whole is, therefore, 705.87 sq. ft. × 2 = 1,411.74 sq. ft., which is equal to 1,411.74 ÷ 100 = 14.12 squares. Ans.

(*b*) The exposed area of a slate is $12 \times 8\frac{1}{2} = 102$ sq. in. In a square there are 144 × 100 = 14,400 sq. in. Since 1 slate covers 102 sq. in., to cover 14,400 sq. in. will require 14,400 ÷ 102 = 141+ slates. Ans.

(33) In the formula for surface of a sphere, $S = \pi d^2$, d is 3 ft.; the surface of the hemisphere is, therefore, $\frac{1}{2}\pi d^2 = \frac{1}{2} \times 3.1416 \times 9 = 14.14$ sq. ft. Deducting 5 sq. ft. for openings, the net surface is 9.14 sq. ft. Ans.

(34) The outer diameter is 10 in. + ($\frac{1}{2}$ in. × 2) = 11 in. The area of an 11-inch circle is 95.03 sq. in.; that of a 10-inch circle is 78.54 sq. in.; the area of the ring is, therefore, 95.03 − 78.54 = 16.49 sq. in. The length is 12 ft. = 144 in.; multiplying 16.49 by 144, the contents are 2,374.56 cu. in. = 2,374.56 ÷ 1,728 = 1.37 cu. ft.; 1.37 cu. ft., at 450 lb. per cu. ft., weigh 1.37 × 450 = 616.5 lb. Ans.

(35) (*a*) The circumference of the tower = πd = 3.1416 × 8 ft. = 25.13 ft. The area of the surface is 25.13 × 28 (the height) = 703.64 sq. ft., which divided by 100, the number of sq. ft. in a square, gives 7.04 squares. Ans.

(*b*) The exposed area of a shingle is 5 × 6 = 30 sq. in. There are 144 sq. in. × 100 = 14,400 sq. in. in a square, which will therefore require 14,400 ÷ 30 = 480 shingles. Ans.

(36) The area of the base plate is 15 × 15 = 225 sq. in., less the area of a 7-inch circle and the portion cut out at the corners, which is equal (for four corners) to the difference between a 4-inch square and a circle 4 inches in diameter; or, 16 sq. in. − 12.57 sq. in. = 3.43 sq. in. The area of a 7-inch circle is 38.48 sq. in. Adding 38.48 and 3.43, the total deduction is 41.91 sq. in., which subtracted from 225 sq. in. leaves 183.09 sq. in. The thickness being $1\frac{1}{2}$ in. = 1.5 in., the contents are 183.09 × 1.5 = 274.63 cu. in.

The height of the cylindrical part is 2 in.; its area is the difference between the areas of an 11-inch and a 10-inch circle, which is $95.03 - 78.54 = 16.49$ sq. in.; hence, the contents are $16.49 \times 2 = 32.98$ cu. in. The triangular side area of the ribs is $\frac{2+0}{2} \times 4\frac{1}{2}$ in. $= 4.5$ in.; the thickness being $\frac{1}{2}$ in., the contents of each are $4.5 \times .5 = 2.25$ cu. in.; and for four the contents are $2.25 \times 4 = 9$ cu. in.

Adding the several portions, the total contents $= 274.63 + 32.98 + 9 = 316.61$ cu. in. The weight is, therefore, $316.61 \times .26 = 82.32$ lb. Ans.

(37) The area of the lower base is $12 \times 30 = 360$ sq. ft.; that of the upper base is $7 \times 22 = 154$ sq. ft. The area of a mid-section is

$$\frac{12+7}{2} \times \frac{30+22}{2} = 9.5 \times 26 = 247 \text{ sq. ft.}$$

By the prismoidal formula, the volume is

$$\tfrac{2}{6}\tfrac{8}{}(360 + 154 + 247 \times 4) = \tfrac{2}{6}\tfrac{8}{} \times 1{,}502 = 7{,}009.33 \text{ cu. ft.}$$
$$= 7{,}009.33 \div 27 = 259.6 \text{ cu. yd.}$$

At \$12.00 per cu. yd., the cost is $259.6 \times 12 = \$3{,}115.20$. Ans.

(38) The volume of the ball is $25 \div .26 = 96.15$ cu. in. The diameter $= \sqrt[3]{\frac{96.15}{.5236}} = \sqrt[3]{183.63} = 5.68$ in. $= 5\frac{11}{16}$ in., nearly. Ans.

(39) The area of the two plates is $2(12 \times \frac{3}{8}) = 9$ sq. in. Each channel consists of a rectangle which has an area of $10 \times \frac{1}{4} = 2\frac{1}{2}$ sq. in., and 2 trapezoids each ($2\frac{1}{2}$ in. $- \frac{1}{4}$ in.) long, and with an average width of $\frac{\frac{5}{8}+\frac{3}{8}}{2} = \frac{1}{2}$ in., and an area of $2 \times (2\frac{1}{4} \times \frac{1}{2}) = 2\frac{1}{4}$ sq. in., making a total for each channel of $2\frac{1}{2} + 2\frac{1}{4} = 4\frac{3}{4}$ sq. in.; for two channels the area is $4\frac{3}{4} \times 2 = 9\frac{1}{2}$ sq. in. Adding the area of the plates, the total is $9\frac{1}{2} + 9 = 18\frac{1}{2}$ sq. in. Multiplying by 12 (inches in a foot), the contents per foot of length are $18\frac{1}{2} \times 12 = 222$ cu. in., and the weight at .283 lb. per cu. in. is $222 \times .283 = 62.83$ lb. Ans.

(40) (*a*) The area of the footing is $\frac{96{,}000}{2{,}000\times 3} = 16$ sq. ft. Each side is therefore $\sqrt{16}$ ft. = 4 ft. Ans.

(*b*) The area of the base plate is $\frac{96{,}000}{2{,}000\times 10} = 4.8$ sq. ft.; and each side is $\sqrt{4.8}$ ft. = 2.19 ft. Ans.

(41) The required area is $\frac{275000}{100000} = 2.75$ sq. in. The diameter $= \sqrt{\frac{2.75}{.7854}} = 1.87$ in. $= 1\frac{7}{8}$ in., nearly.

(42) The monument consists of a frustum of a pyramid and a pyramid. For the former, the area of the lower base is $3\times 3 = 9$ sq. ft.; that of the upper base is $1\times 1 = 1$ sq. ft. Using the formula

$$V = \frac{h}{3}(A + a + \sqrt{A\times a}) = \tfrac{1}{3}\tfrac{8}{}(9+1+\sqrt{9\times 1}) = 78 \text{ cu. ft.}$$

The volume of the pyramid is $\frac{a\,h}{3}$; $a = 1$ sq. ft.; $h = 2$ ft.; therefore, $\frac{a\,h}{3} = \frac{1\times 2}{3} = \frac{2}{3}$ cu. ft. The total is $78 + \frac{2}{3}$ $= 78\frac{2}{3}$ cu. ft., and the weight at 170 lb. per cu. ft., is $78\frac{2}{3}\times 170 = 13{,}373\frac{1}{3}$ lb. Ans.

8-8

ARCHITECTURAL ENGINEERING.

(QUESTIONS 1-72. SEC. 5.)

(1) At the point of support. See Art. **88** and formula **10,** Art. **97.**

(2) 120 pounds per square foot. See Table 4, Art. **42.**

(3) The depth of the beam in inches should not be less than half the span of the beam in feet. See Art. **109.**

(4) Beginning at the point *a*, Fig. 1, draw the line *a b* parallel to the force of 800 pounds, and on this line lay off the distance *a b* equal to 800 pounds to the assumed scale. From *b* draw *b c* parallel to the force of 500 pounds and lay off on it the distance *b c* equal to 500 pounds to scale. From *c* draw *c d* parallel and, to the assumed scale, equal to the force of 1,900 pounds. In the same way, draw *d e* and *e f* equal, respectively, to the forces of 1,000 pounds and 1,600 pounds, thus locating the point *f*. Join *f* and *a*, and *a f*, measured by the scale with which the sides from *a* to *f* were laid off, is the magnitude of the resultant of the several forces, and the direction of the resultant, as given by the arrow on *a f*, is opposite to the general direction in which the given forces were laid off.

(5) The horizontal component of a force is that part of the force which tends to produce motion in a horizontal

direction. The vertical component of a force is that part of the force which tends to produce motion in a vertical direction. See Art. **19.**

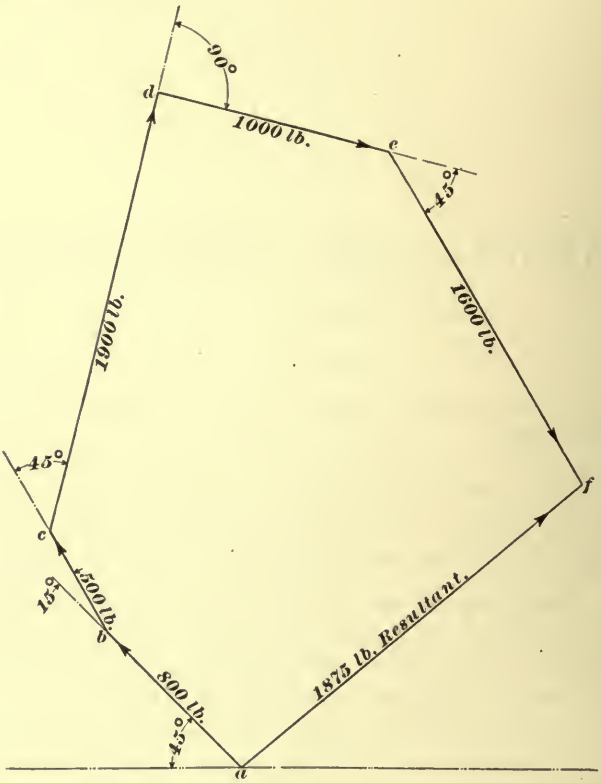


FIG. 1.

(6) In accordance with the principles given in Art. **111,** the weight to be supported is that of a triangular piece of brickwork whose volume is $\frac{1}{2} \times 25 \times \frac{2.5}{3} \times 2 = 208\frac{1}{3}$ cu. ft. The weight of a cubic foot of brickwork in lime mortar, from Table 1, Art. **40,** is 120 pounds; the total weight to be supported by the beams is, therefore, $208\frac{1}{3} \times 120 = 25{,}000$ pounds. The bending moment, from formula **14,** Art. **97,** is

$$M = \frac{25{,}000 \times 25 \times 12}{6} = 1{,}250{,}000 \text{ in.-lb.}$$

Since the safe unit stress is 15,000 pounds per square inch, the section modulus for the two beams must be $1,250,000 \div 15,000 = 83\frac{1}{3}$, and the section modulus for each beam, $83\frac{1}{3} \div 2 = 41\frac{2}{3}$. Referring to Table 11, we find the section modulus of a 12-inch 39.4-pound beam to be 44.72, and of a 15 inch 41.2-pound beam, 57.73, and that these are the lightest beams that will satisfy the requirements; consequently, either of these may be used. Ans.

(7) See Art. **110.**

(8) The moments of the loads with respect to the right-hand support as a center are as follows:

$$\begin{aligned} 4,500 \times\ 9 &= \quad 4\,0,5\,0\,0 \text{ ft.-lb.} \\ 8,100 \times 14 &= 1\,1\,3,4\,0\,0 \text{ ft.-lb.} \\ 6,000 \times 20 &= 1\,2\,0,0\,0\,0 \text{ ft.-lb.} \\ \text{Total,} \quad &\quad 2\,7\,3,9\,0\,0 \text{ ft.-lb.;} \end{aligned}$$

the reaction R_1 is, therefore, $273,900 \div 32 = 8,559$ pounds, nearly. Beginning at the left-hand support, where the reaction is 8,559 pounds, and subtracting from the reaction the

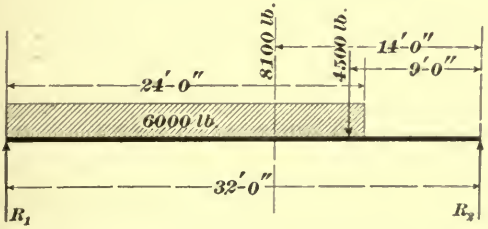


FIG. 2.

successive loads towards the right, we find that up to the point of application of the load of 8,100 pounds, 18 feet from the left support, the load is $18 \times \frac{6000}{24} = 4,500$ pounds; this is less than the reaction by $8,559 - 4,500 = 4,059$ pounds, but, when the concentrated load is considered, the loads exceed the reaction. The shear, therefore, changes sign under the load of 8,100 pounds, 18 feet from the left-hand end. Ans.

(9) In office buildings it is not likely that all the floors will be fully loaded at any one time, but in storage warehouses all the floors are often fully loaded. See Art. **66.**

(10) From Table 6, Art. **61,** the ultimate compressive strength of yellow pine is 4,400 pounds per square inch; assuming in accordance with the last paragraph of Art. **70,** a column 10 inches square and substituting in formula **3,** Art. **70,** the ultimate compressive strength of the column is

$$S = 4{,}400 - \frac{4{,}400 \times 216}{100 \times 10} = 3{,}449.6 \text{ lb. per sq. in.}$$

of sectional area. With a factor of safety of 5, the safe load on this column is $\frac{3{,}449.6}{5} \times 100 = 68{,}992$ pounds, much less than the load to be carried. Assuming, now, a column 12 inches square, we have

$$S = 4{,}400 - \frac{4{,}400 \times 216}{100 \times 12} = 3{,}608 \text{ lb. per sq. in,}$$

and the safe load is $\frac{3608}{5} \times 144 = 103{,}910$ pounds; the size of the column should, therefore, be 12 in. $\times$ 12 in. Ans.

(11) Taking the center of moments over the left reaction, the moments of the loads are as follows:

$4{,}000 \times 5 =$	2 0,0 0 0 ft.-lb.
$2{,}000 \times 10 =$	2 0,0 0 0 ft.-lb.
$3{,}000 \times 12 =$	3 6,0 0 0 ft.-lb.
Total,	7 6,0 0 0 ft.-lb.

The reaction R_2 is $76{,}000 \div 20 = 3{,}800$ pounds; subtracting from this the load b (see Art. **85**), the shear between the points b and c is $3{,}800 - 3{,}000 = 800$ pounds. Ans.

(12) Taking the center of moments over R_1, the moments of the loads are as follows:

$9{,}000 \times 15 =$	1 3 5,0 0 0 ft.-lb.
$11{,}000 \times 25 =$	2 7 5,0 0 0 ft.-lb.
$19{,}000 \times 28 =$	5 3 2,0 0 0 ft -lb.
$10{,}000 \times 50 =$	5 0 0,0 0 0 ft.-lb.
Total,	1,4 4 2,0 0 0 ft.-lb.

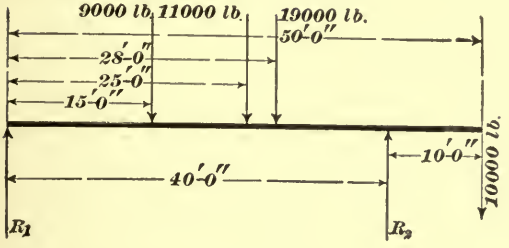


FIG. 3.

The reaction R_2 is $1{,}442{,}000 \div 40 = 36{,}050$ pounds. Ans.

(13) See Art. **78.**

(14) See Art. **75.**

(15) The brackets under the **I** beams are too thin, and the webs under these brackets do not correspond with the requirements stated in Art. **74**; there is also no provision for bolting the **I** beams to the column. The base is too thin and has no ribs for strengthening it. The column should be designed according to the principles laid down in Art. **74.**

(16) The sum of the reactions must equal the sum of the loads. See Art. **78.**

(17) The sum of the loads is $8{,}000 + 7{,}000 + 9{,}000 = 24{,}000$ pounds; in accordance with the principles of Art. **78,** since one reaction is 12,000 pounds, the other must be $24{,}000 - 12{,}000 = 12{,}000$ pounds. Ans.

(18) Applying formula **19,** Art. **113,** and substituting the value of the coefficient c for granite, the safe load the lintel will carry is

$$W = \frac{25 \times 20^2}{60} \times .12 = 20 \text{ tons.} \quad \text{Ans.}$$

(19) In accordance with the principles stated in Art. **116,** the stress in each of the struts is $\frac{11}{30}$ of the total uniformly distributed load on the beam, that is, $1{,}000 \times 30 \times \frac{11}{30} = 11{,}000$ pounds.

(*a*) The length of the diagonal portion of the camber rod is $\sqrt{10^2+2^2} = 10.198$ feet; applying formula **24,** Art. **116,** the stress on the camber rod is

$$\frac{10.198}{2} \times 11{,}000 = 56{,}089 \text{ pounds.} \quad \text{Ans.}$$

(*b*) Applying formula **26,** the stress in the beam is

$$\tfrac{10}{2} \times 11{,}000 = 55{,}000 \text{ pounds.} \quad \text{Ans.}$$

(20) The allowable unit stress is $60{,}000 \div 3 = 20{,}000$ pounds; the safe total stress is $6 \times 20{,}000 = 120{,}000$ pounds. Ans.

(21) The compressive stress per square inch of area is $135{,}360 \div 144 = 940$ pounds. From Table 6, Art. **61,** the ultimate compressive strength of oak is 3,600 pounds per square inch; the factor of safety is, therefore, $3{,}600 \div 940 = 3.83$. Ans.

(22) A factor of safety of at least 10 should be used. See Art. **113.**

(23) When all the forces acting in any one direction are balanced by an equal set of forces acting in the opposite direction. See Art. **78.**

(24) The total load on the girder is $2{,}500 \times 30 = 75{,}000$ pounds.

(*c*) According to the principles laid down in Art. **115,** the stress in the central strut is

$$W = \tfrac{5}{8} \times 75{,}000 = 46{,}875 \text{ lb.} \quad \text{Ans.}$$

(*a*) The length of one-half of the camber rod is $\sqrt{15^2+2^2} = 15.13275$; applying formula **21,** Art. **115,** the stress in the camber rod is

$$\frac{15.13275}{2} \times \frac{46{,}875}{2} = 177{,}337 \text{ lb.} \quad \text{Ans.}$$

(*b*) Applying formula **22,** the compressive stress in the beam is

$$\tfrac{15}{2} \times \tfrac{46875}{2} = 175{,}781 \text{ lb.} \quad \text{Ans.}$$

(25) Referring to Table 2, Art. **40,** the weights of the various materials in the covering of the roof are found to be as follows:

Spanish tiles, 1 sq. ft.,	$8\frac{1}{2}$ lb.
3 inches spruce @ 2 lb., 1 sq. ft.,	6 lb.
2 layers Neponset roofing felt, 1 sq. ft.,	$\frac{1}{2}$ lb.
Total,	15 lb. Ans.

(26) The dead load comprises the weight of the building with its permanent fixtures, while the live load consists of the weight of people, furniture, movable stocks of goods, and all materials or fixtures subject to frequent changes of position. See Arts. **40** and **42.**

(27) Since the metal is $\frac{3}{4}$ inch thick, the inner diameter is $10-2\times\frac{3}{4}=8\frac{1}{2}$ inches. Applying formula **9,** Art. **73,** we have

$$R^2=\frac{10^2+8.5^2}{16}=10.77. \quad \text{Ans.}$$

(28) Applying formula **15,** Art. **101,** we have the section modulus

$$K=\frac{1\times12^2}{6}=24;$$

then, by the definition, Art. **99,** the resisting moment is $24\times10{,}000=240{,}000$ in.-lb. Ans.

(29) A sketch of the construction is shown in Fig. 4. We

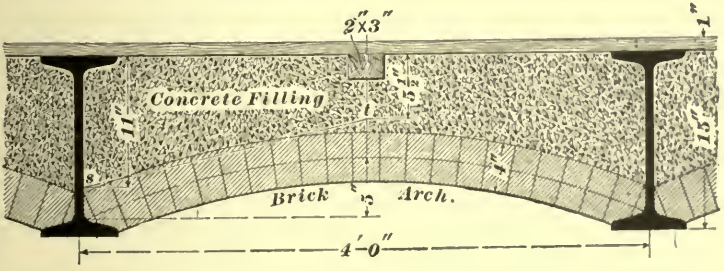


FIG. 4.

will first compute the weight of a section of the brick arch

1 foot long. The area of the section of the arch is equal to the product of the length of its center line multiplied by the thickness; from the formula $l = \frac{4\sqrt{c^2 + 4h^2} - c}{3}$ (see *Geometry and Mensuration*), in which l = length of arc, c = length of chord, and h = rise, by assuming the length of chord to be the distance from center to center of beams and the rise of the center line of the arch to be 5 inches, the given rise of the arch, we have the length of the center line of the arch

$$l = \frac{4\sqrt{48^2 + 4 \times 5^2} - 48}{3} = 49 \text{ inches, nearly;}$$

the area of the section is, therefore, $\frac{49 \times 4}{144} = 1.36$ sq. ft., and the volume of brickwork per foot of length of the beams is $1.36 \times 1 = 1.36$ cu. ft. Since the weight of a cubic foot of brickwork in cement is 130 pounds per cu. ft. (see Table 1, Art. **40**), the weight of brick per square foot of floor area is

$$\frac{1.36 \times 130}{4} = 44.2 \text{ pounds.}$$

An approximation to the volume of a section of the concrete 1 foot long, near enough for the present purpose, is the following: Draw a straight line $s\,t$, Fig. 4, which cuts the upper part of the arch in such a manner that the area of the segment of the arch above the line is, approximately, equal to the section of the concrete between the line and the top of the arch. The ends of the line $s\,t$, drawn in this manner, are, respectively, 11 inches and $5\frac{1}{2}$ inches below the top of the concrete and the area of the section included between this line and the top, neglecting the area of the beam, is $\frac{11 + 5\frac{1}{2}}{2} \times 24 = 198$ square inches; the area of the whole section of concrete is, therefore, $\frac{198 \times 2}{144} = 2.75$ sq. ft., giving a volume of $2.75 \times 1 = 2.75$ cu. ft. of concrete per foot of length of the beams. The weight of concrete per cubic foot, from Table 1, is 140 pounds; the weight per

square foot of floor area is, therefore, $\frac{2.75 \times 140}{4} = 96.25$ pounds.

In Table 2, Art. **40,** the weight of a square foot of yellow pine 1 inch thick is found to be 4 pounds.

The total dead load, per square foot of floor area, is $44.2 + 96.25 + 4 = 144.45$ pounds, and the total dead and live load to be carried by a single beam is $(144.45 + 200) \times 23 \times 4 = 31{,}689.4$ pounds.

Applying formula **13,** Art. **97,** the bending moment is found to be

$$M = \frac{31{,}689.4 \times 23 \times 12}{8} = 1{,}093{,}284 \text{ in.-lb.}$$

Dividing by the allowable unit stress, the section modulus required is $1{,}093{,}284 \div 15{,}000 = 72.9$; by referring to Table 11, we find that a 15-inch 56.9-pound beam has a section modulus of 74.77, and that this is the lightest beam that will satisfy the required conditions. Ans.

(30) The sum of the moments of the given loads, with respect to the support as a center, is $60 \times 15 + 80 \times 6 = 1{,}380$ ft.-lb. Since the lever arm of the required load is 15 feet, the load must be $1{,}380 \div 15 = 92$ pounds. Ans.

(31) (*a*) See Art. **90.**

(*b*) See Art. **89.**

(32) 40 pounds per square foot of floor area. See Art. **42.**

(33) Taking the center of moments over the right reaction, the moments of the loads are as follows:

$8{,}000 \times 10 =$	80,000 ft.-lb.
$10{,}000 \times 18 =$	180,000 ft.-lb.
$7{,}200 \times 25 =$	180,000 ft.-lb.
$1{,}000 \times 20 \times (35 - 10) =$	500,000 ft.-lb.
Total,	940,000 ft.-lb.

The reaction R_1 is $940{,}000 \div 35 = 26{,}857\frac{1}{7}$ pounds; and the reaction R_2 is $8{,}000 + 10{,}000 + 7{,}200 + 1{,}000 \times 20 - 26{,}857\frac{1}{7} = 18{,}342\frac{6}{7}$ pounds. Ans.

(34) Under the load at which the shear changes sign. See Art. **96.**

(35) See Art. **72.**

(36) The total load on the beam is $W = 8 \times 500 = 4{,}000$ pounds; by applying formula **11,** Art. **97,** the maximum bending moment is found to be

$$M = \frac{4{,}000 \times 8}{2} = 16{,}000 \text{ ft.-lb.} \quad \text{Ans.}$$

(37) Assuming, according to Table 4, Art. **42,** a load of 150 pounds per square foot of floor area, the entire live load will be $60 \times 290 \times 150 = 2{,}610{,}000$ lb. Ans.

(38) Since the beam is uniformly loaded, each reaction is equal to 45,000 lb., one-half the load; according to Art. **86,** the maximum shear is, also, 45,000 lb. Ans.

(39) See Arts. **84** and **85.**

(40) See Arts. **63** and **64.**

(41) The inside diameter of the column is $6 - 2 \times \frac{3}{4} = 4\frac{1}{2}$ inches; applying formula **9,** Art. **73,** the square of the radius of gyration of the section is

$$R^2 = \frac{6^2 + 4.5^2}{16} = 3.516.$$

From Table 6, Art. **61,** the ultimate compressive strength of cast iron is 81,000 pounds per square inch; the length of the column is $10 \times 12 = 120$ inches. Substituting these values in formula **4,** Art. **73,** the ultimate unit strength of the column is

$$S = \frac{81{,}000}{1 + \frac{120^2}{3{,}600 \times 3{,}516}} = 37{,}895 \text{ lb. per sq. in.}$$

With a factor of safety of 6, the safe stress per sq. in. is $37{,}895 \div 6 = 6{,}316$ pounds. The area of the section is $6^2 \times .7854 - 4.5^2 \times .7854 = 12.37$ square inches; multiplying by the safe unit stress, the safe load is $12.37 \times 6{,}316 = 78{,}000$ lb., in round numbers. Ans.

(42) Applying formula **1**, Art. **55**, the unit stress is

$$S = \frac{80{,}000}{5} = 16{,}000 \text{ lb.} \quad \text{Ans.}$$

(43) The sectional area of the pier is $3 \times 4 \times 144 = 1{,}728$ square inches; since its height is less than 16 times its thickness (see Art. **65**), the value for the compressive strength of brick in lime mortar given in Table 8 applies, and the pier will safely support a load of $1{,}728 \times 100 = 172{,}800$ lb. Ans.

(44) (*a*) Taking the center of moments over the right support, the moments of the loads are as follows:

$$\begin{aligned} 1{,}000 \times 12 \times 6 &= 72{,}000 \text{ ft.-lb.} \\ 2{,}000 \times 8 \times 16 &= 256{,}000 \text{ ft.-lb.} \\ 1{,}000 \times 4 \times 22 &= 88{,}000 \text{ ft.-lb.} \\ \text{Total,} \qquad & 416{,}000 \text{ ft.-lb.} \end{aligned}$$

The left reaction is $416{,}000 \div 24 = 17{,}333\frac{1}{3}$ pounds; beginning at the left support and subtracting the successive loads, we find that the first uniformly distributed load is less than the left reaction, but that the sum of the first two uniformly distributed loads is greater; the shear, therefore, changes sign at some point under the load of 2,000 pounds per foot. To find this point we have $17{,}333\frac{1}{3} - 4 \times 1{,}000 = 13{,}333\frac{1}{3}$, and $13{,}333\frac{1}{3} \div 2{,}000 = 6\frac{2}{3}$ feet, the distance from the beginning of the second uniformly distributed load at which the shear changes sign. (See Art. **87**.) Since the greatest bending moment is at the section where the shear changes sign (see Art. **96**), the center about which the greatest bending moment is to be calculated is at a distance of $4 + 6\frac{2}{3} = 10\frac{2}{3}$ feet from the left support, and the magnitude of the moment is $17{,}333\frac{1}{3} \times 10\frac{2}{3} - (4{,}000 \times 8\frac{2}{3} + 13{,}333\frac{1}{3} \times 3\frac{1}{3}) = 105{,}800$ ft.-lb., in round numbers. Ans.

(*b*) The distance of the section where the shear changes sign from the right-hand end is $24 - 10\frac{2}{3} = 13\frac{1}{3}$ ft. = 13 ft. 4 in. Ans.

(45) This problem must be solved by a series of approximations; see last paragraph of Art. **70**. By substituting a

thickness of metal less than 1 inch in formula **9,** Art. **73,** and applying the resultant value of R^2 to formula **4,** it is found that the unit stress in the column is greater than that allowed by the prescribed factor of safety; on the other hand, if a thickness greater than 1 inch is used, the unit stress is less than the allowable stress and more metal is used than is required. Using a thickness of 1 inch, the inner diameter is $16-2 = 14$ inches, and by formula **9,** the square of the radius of gyration is

$$R^2 = \frac{16^2+14^2}{16} = 28.25.$$

The length of the column is $24\times12 = 288$ inches, and from Table 6, Art. **61,** the ultimate compressive strength of cast iron is 81,000 pounds per square inch; substituting in formula **4,** Art. **73,** the ultimate strength of the column is

$$S = \frac{81{,}000}{1+\frac{288^2}{3{,}600\times28.25}} = 44{,}600 \text{ lb. per sq. in.},$$

and the allowable unit stress, $44{,}600\div5 = 8{,}920$ lb. per sq. in. The area of the section is $16^2\times.7854-14^2\times.7854 = 47.12$ sq. in.; the column will, therefore, safely carry $47.12\times8{,}920 = 420{,}310$ lb., which is practically the amount required, and a thickness of metal of 1 inch is sufficient. Ans.

(46) In Table 8, Art. **65,** the safe bearing value of stiff clay is found to be 2.5 tons = 5,000 lb. per sq. ft.; therefore, the footing will safely carry a load of $5{,}000\times25 = 125{,}000$ lb. Ans.

(47) At the section midway between the ends. See Art. **96.**

(48) The general method of drawing this diagram is very similar to that described in Art. **131,** and the student should experience no difficulty in understanding the construction shown in Figs. 5 and 6. Since the truss and its loading are symmetrical, the stresses in the corresponding members of the two halves are equal, and the diagram for but one-half need be drawn.

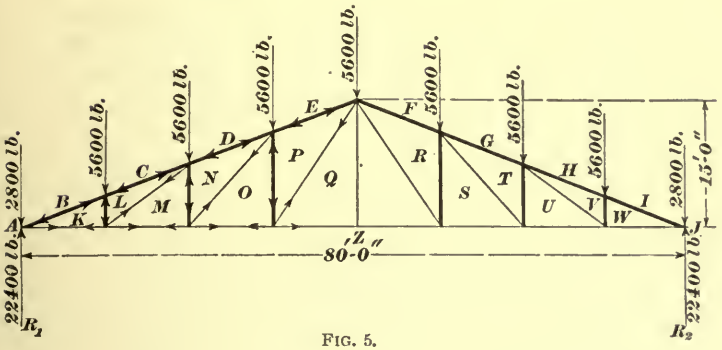


FIG. 5.

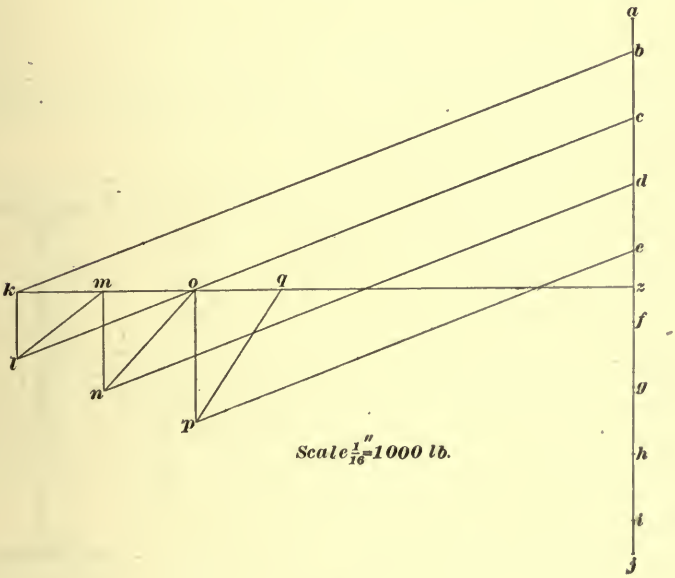


FIG. 6.

The stresses in the several members of the left-hand half as determined by the diagram, are as follows:

Member.	Stress. Lb.	Kind of Stress.
B K	55,800	Compressive.
C L	55,800	Compressive.

Member.	Stress. Lb.	Kind of Stress.
DN	48,000	Compressive.
EP	40,000	Compressive.
KL	5,600	Compressive.
LM	9,300	Tensile.
MN	8,400	Compressive.
NO	11,200	Tensile.
OP	11,200	Compressive.
PQ	13,200	Tensile.
KZ	52,300	Tensile.
MZ	45,000	Tensile.
OZ	37,300	Tensile.
QZ	30,000	Tensile.

(49) The total uniformly distributed load is $2{,}000 \times 30 = 60{,}000$ pounds, and the length of the beam is $30 \times 12 = 360$ inches; substituting in formula **13,** Art. **97,** the greatest bending moment is

$$M = \frac{60{,}000 \times 360}{8} = 2{,}700{,}000 \text{ in.-lb.} \quad \text{Ans.}$$

(50) Applying formula **15,** Art. **101,** the section modulus is

$$K = \frac{12 \times 16^2}{6} = 512.$$

The modulus of rupture of yellow pine, from Table 6, Art. **61,** is 7,300 pounds per square inch; the working stress is, therefore, $7{,}300 \div 4 = 1{,}825$ lb. per sq. in. Applying formula **16,** Art. **102,** and letting S represent the unit working stress, the safe load is

$$W = \frac{\left(\frac{512 \times 1{,}825}{12}\right)8}{30} = 20{,}800 \text{ lb. in round numbers.} \quad \text{Ans.}$$

(51) Referring to Table 2, Art. **40,** the weight of the slate roof is

$\frac{3}{16}$-inch slate	$6\frac{3}{4}$ lb. per sq. ft.
1-inch hemlock	$2\frac{1}{2}$ lb. per sq. ft.
2 layers roofing felt	$\frac{1}{2}$ lb. per sq. ft.
Total	$9\frac{3}{4}$ lb. per sq. ft.

The weight of the slag roof is

4-ply slag	4 lb. per sq. ft.
2-inch spruce sheathing...	4 lb. per sq. ft.
Total.....	8 lb. per sq. ft.

The difference in favor of the slag roof is $9\frac{3}{4} - 8 = 1\frac{3}{4}$ lb. per sq. ft. Ans.

(52) In Table 8, Art. **65,** it is stated that the safe bearing value of brickwork in Rosendale cement mortar is 150 pounds per square inch; the capstone must, therefore, have an area of $1{,}000{,}000 \div 150 = 6{,}667$ sq. in. The length of the side of a square whose area is 6,667 sq. in. is $\sqrt{6{,}667} = 82$ in., nearly = 6 ft. 10 in. Ans.

(53) The connection, which is shown in Fig. 7, is designed according to the standard given in Art. **112.**

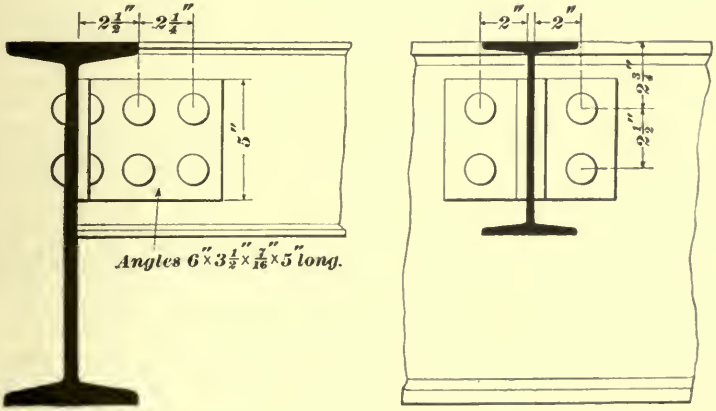


FIG. 7.

(54) Applying formula **17,** Art. **105,** the section modulus is $K_t = \frac{14 \times 12}{3.2} = 52.5$. Ans.

(55) Using the value for the ultimate compressive strength of spruce given in Table 6, Art. **61,** and applying formula **3,**

Art. **70,** the ultimate compressive strength of the column is

$$S = 3{,}000 - \frac{3{,}000 \times 120}{100 \times 4} = 2{,}100 \text{ lb. per sq. in.}$$

With a factor of safety of 4, the safe unit stress is $2{,}100 \div 4 = 525$ lb. per sq. in., and the allowable load is $525 \times 4 \times 12 = 25{,}200$ lb. Ans.

(56) Since the rise is 6 inches per foot of span, the total rise is $50 \times \frac{1}{2} = 25$ feet; the length of each rafter is $\sqrt{50^2 + 25^2} = 55.9$ ft., and the panel length is $55.9 \div 4 = 13.975$ ft. The area of each panel is $13.975 \times 20 = 279.5$ sq. ft.; from Table 5, Art. **44,** the wind pressure normal to the slope, for a rise of 6 inches per foot, is 23.7 pounds; the panel wind load is, therefore, $279.5 \times 23.7 = 6{,}624$ lb. Ans.

(57) In accordance with Table 8, Art. **65,** the safe bearing value of the soil under the pier is 2.5 tons $= 2.5 \times 2{,}000 = 5{,}000$ lb. per sq. ft.; the concrete base must, therefore, cover an area of $200{,}000 \div 5{,}000 = 40$ sq. ft.; this area will be provided by a base 6 ft. 4 in. square.

In Table 8 the bearing value of rubble masonry in Portland cement mortar is stated to be 150 pounds per square inch; this demands a capstone with an area of $200{,}000 \div 150 = 1{,}333\frac{1}{3}$ square inches, which will be provided by the use of a stone 3 ft. 1 in. square. Since the bearing values of the concrete and the masonry resting on it are each 150 pounds per square inch, sufficient bearing strength would be provided if the base of the masonry were 3 ft. 1 in. square, but, according to Art. **66,** the concrete offset should not be more than 8 inches, which requires that the base of the masonry be 6 ft. 4 in. $- 2 \times 8$ in. $= 5$ ft. 0 in. square.

From the calculated dimensions and the general instructions given in Arts. **66** and **67,** a drawing of the pier similar to Fig. 38, Art. **67,** can now be made.

(58) Applying formula **2,** Art. **57,** the unit strain is

$$s = \frac{\frac{3}{16}}{15 \times 12} = .00104 \text{ in.} \quad \text{Ans.}$$

(59) Applying formula **12**, Art. **97**, the bending moment due to the concentrated load is

$$M = \frac{4{,}000 \times 300}{4} = 300{,}000 \text{ in.-lb.};$$

and, by formula **13**, the bending moment due to the uniformly distributed load is

$$M = \frac{500 \times 25 \times 300}{8} = 468{,}750 \text{ in.-lb.};$$

the total bending moment is, therefore, $300{,}000 + 468{,}750 = 768{,}750$ in.-lb. The section modulus of the beam must be $768{,}750 \div 15{,}000 = 51.25$ (see Art. **104**); by referring to Table 11, it is seen that the lightest beam that will fill the conditions is a 15-inch 41.2-pound beam. Ans.

(60) Assuming a bar with a sectional area of 1 square inch, the weight per foot of length is $12 \times .283 = 3.396$ pounds (see Table 1, Art. **40**); the length required is, therefore, $60{,}000 \div 3.396 = 17{,}668$ ft. Ans.

(61) (*a*) See Art. **55.**

(*b*) See Art. **57.**

(62) See Art. **11.**

(63) (*a*) Applying formula **8**, Art. **73**, the square of the radius of gyration of the section is

$$R^2 = \frac{10^2 + 8^2}{12} = 13.67;$$

by formula **4**, Art. **73**, the ultimate strength per square inch of the column is

$$S = \frac{81{,}000}{1 + \frac{240^2}{3{,}600 \times 13.67}} = 37{,}320 \text{ lb., nearly.}$$

With a factor of safety of 5, the safe unit stress is $37{,}320 \div 5 = 7{,}464$ pounds; the area of the section is $10^2 - 8^2 = 36$ square inches, which makes the load the column will safely carry $7{,}464 \times 36 = 268{,}700$ lb. in round numbers. Ans.

(*b*) The footing must have an area of $268{,}700 \div 5{,}000 = 53.74$ sq. ft.; a footing 7 ft. 4 in. square will furnish the required area.

The bearing value of the brick is 200 pounds per square inch. (See Table 8, Art. **65.**) The capstone must, therefore, have an area of $268{,}700 \div 200 = 1{,}344$ square inches; a stone 3 ft. 1 in. square has an area of 1,369 square inches, and this size will be used. The bearing value of the concrete is 150 pounds per square inch (see Table 8); therefore, in order to safely carry the load, the base of the brickwork must have an area of $268{,}700 \div 150 = 1{,}791$ square inches; this corresponds to a square whose side is a little more than 3 ft. 6 in., which would make the offset of the concrete (7 ft. 4 in. $-$ 3 ft. 6 in.) $\div 2 = 1$ ft. 11 in., a value greater than is permissible. Allowing the maximum offset, the base of the masonry becomes 7 ft. 4 in. $- 2 \times 8$ in. $= 6$ ft. square.

The safe bearing value of the limestone cap is 500 pounds per square inch (see Table 8), which makes the area required for the base of the column $268{,}700 \div 500 = 538$ square inches, corresponding to a square each of whose sides is 23.19, say 24 inches.

Making the thickness of the capstone 18 inches, practically one-half of the length of a side, the permissible offset for the base is $18 \div 2 = 9$ inches; a 24-inch base on the 3′ 1″ stone gives an offset of $(37 - 24) \div 2 = 6\frac{1}{2}$ in., which is well within the prescribed limits.

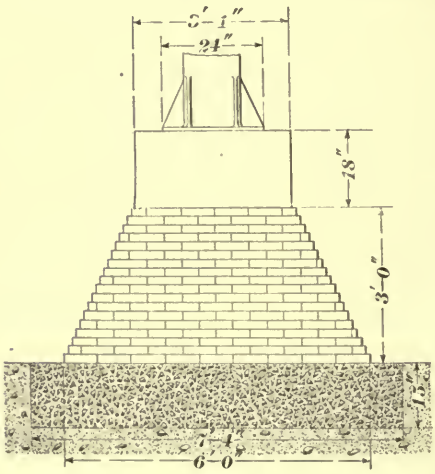
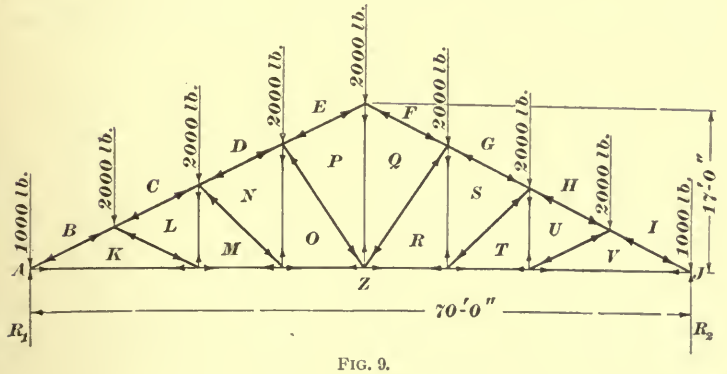


FIG. 8.

The general features of the pier, designed in accordance with the above calculations and the principles laid down in Arts. **66** and **67,** are shown in Fig. 8.


2000 lb.
2000 lb.
2000 lb.
2000 lb.
2000 lb.
2000 lb.
2000 lb.
1000 lb.
1000 lb.
17'-0"
A
B
C
D
E
F
G
H
I
J
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L
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P
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R
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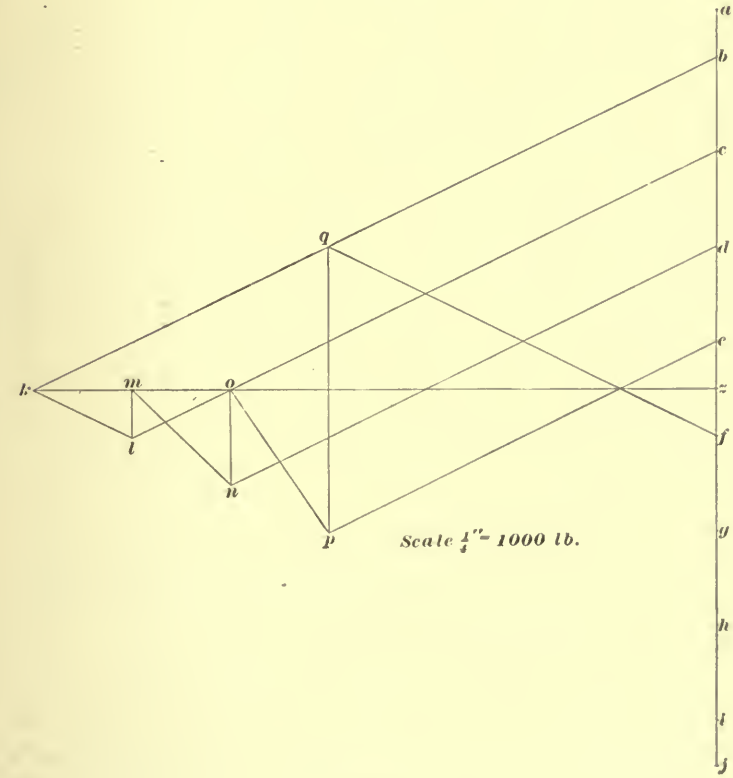
T
U
V
Z
70'-0"
R1
R2

FIG. 9.


a
b
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Scale 1/4" = 1000 lb.

FIG. 10.

(64) Fig. 9 is the frame diagram and Fig. 10 the stress diagram for the dead load, and Figs. 11 and 12 are the

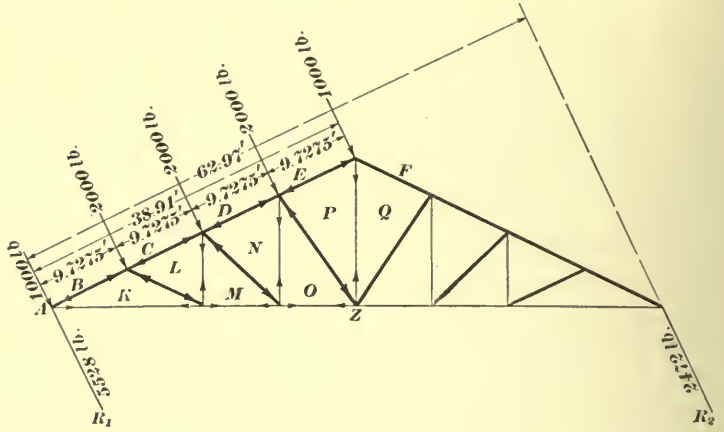

Fig. 11.

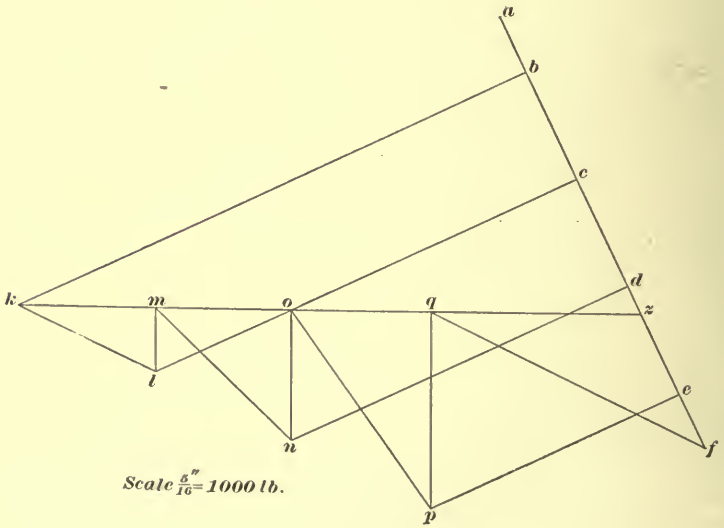

Fig. 12.

corresponding diagrams for the wind load; the construction in each case is exactly like that explained in Art. **131,** and no

further explanation is necessary. The stresses, as determined by the diagrams, Figs. 10 and 12, are as follows:

Member.	Dead-Load Stress. Lb.	Wind-Load Stress. Lb.	Kind of Stress.
BK.....	16,100	9,300	Compressive
CL.....	13,800	7,800	Compressive.
DN.....	11,500	6,200	Compressive.
EP.....	9,200	4,600	Compressive.
KL	2,300	2,500	Compressive.
MN.....	2,900	3,200	Compressive.
OP.....	3,600	4,000	Compressive.
LM	1,000	1,100	Tensile.
NO	2,000	2,200	Tensile.
PQ	6,000	3,300	Tensile.
KZ	14,500	10,400	Tensile.
MZ.....	12,400	8,100	Tensile.
OZ	10,400	5,800	Tensile.
FQ.....	9,200	5,100	Compressive.

(65) Since the moment is the product of the magnitude of the force into its lever arm, the length of the lever arm is found by dividing the moment by the magnitude of the force; in the present case, lever arm $= 275 \div 25 = 11$ ft. Ans.

(66) See Arts. **94** to **96.**

(67) The deflection should not exceed $\frac{1}{360}$ of an inch for each foot of space. (See Art. **118.**)

(68) (*a*) and (*b*) 3 to 4. (See Table 7, Art. **64.**)

(69) The area of the section is $2 \times \frac{1}{4} = \frac{1}{2}$ sq. in.; the ultimate tensile strength is $60{,}000 \times \frac{1}{2} = 30{,}000$ lb. Ans.

(70) From the frame diagram, Fig. 13, construct the stress diagram, Fig. 14. The truss and its loads are each symmetrical, making it unnecessary to draw more than the upper half of the stress diagram; the construction is so similar to that described in Arts. **131** and **132** that no explanation

is needed. The student should note, however, that the distance ef on the load line represents a load of 3,800 + 2,000 = 5,800 pounds, being correspondingly longer than the distances bc, cd, de, fg, etc., which represent loads of but 3,800 pounds.

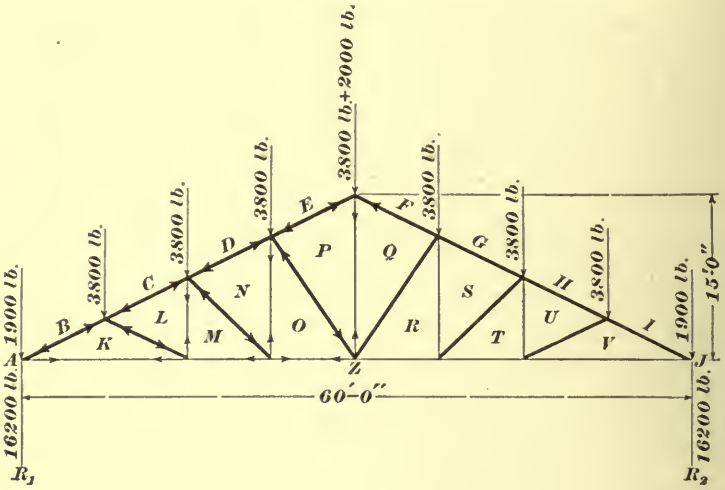

FIG. 13.

The stresses in the several members, as determined by the stress diagram, Fig. 14, are as follows:

Member.	Stress. Lb.	Kind of Stress.
BK	32,000	Compression.
CL	27,800	Compression.
DN	23,500	Compression.
EP	19,300	Compression.
KL	4,200	Compression.
MN	5,400	Compression.
OP	6,800	Compression.
LM	1,900	Tension.
NO	3,800	Tension.
PQ	11,400	Tension.
KZ	28,600	Tension.

Member.	Stress. Lb.	Kind of Stress.
MZ	24,900	Tension.
OZ	21,000	Tension.
QF	19,300	Compression.

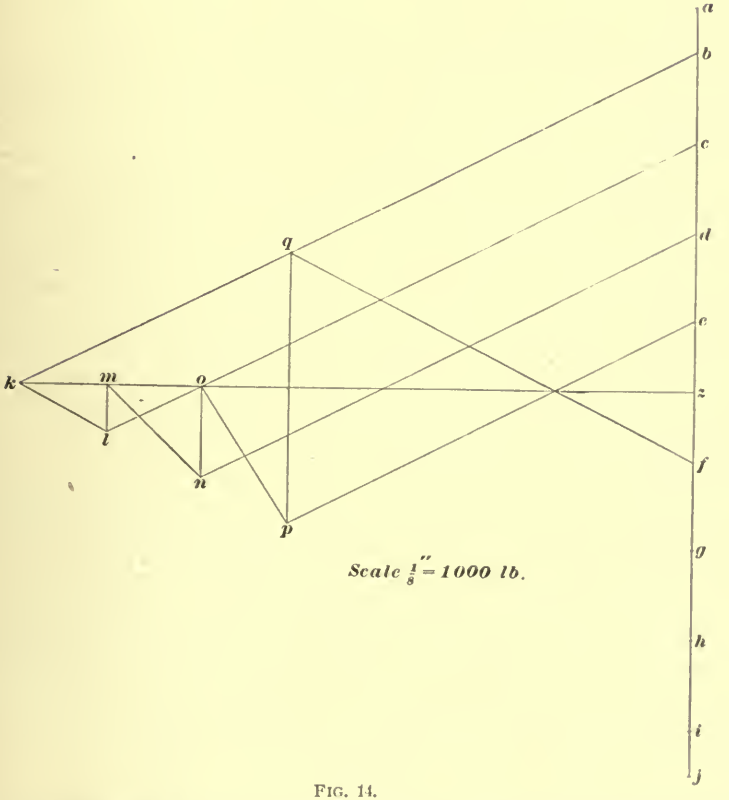

FIG. 14.

Considering first the size of timber required for the rafter member, we find that it must be proportioned to withstand a compressive stress of 32,000 pounds, the stress in the member BK. The unsupported length of the member is 8 ft. $4\frac{1}{2}$ in. $= 100\frac{1}{2}$ in., and the ultimate compressive strength of yellow pine, from Table 6, Art. **61,** is 4,400 pounds per square inch; assuming a timber whose least dimension is 6 inches

and substituting in formula **3,** Art. **70,** the ultimate compressive strength of the rafter is

$$S = 4{,}400 - \frac{4{,}400 \times 100\frac{1}{2}}{100 \times 6} = 3{,}663 \text{ lb. per sq. in.};$$

with a factor of safety of 5, the unit stress is $3{,}663 \div 5 = 733$ lb. per sq. in., nearly, and the area of the section required is $32{,}000 \div 733 = 44$ sq. in., nearly. A $6'' \times 8''$ timber is the smallest even size that will provide the area demanded by the conditions.

The compression member OP, as determined by scaling from the frame diagram, is about 13 ft. 6 in. long, and the stress in it is 6,800 pounds; a $4'' \times 6''$ timber is amply strong for this member, and, by placing the rafter with its long side vertical, will present a surface flush with that of the rafter. The compression members MN and KL will not be designed for strength, as that would prescribe a size too small to be practicable, but will be made of either $3'' \times 6''$ or $4'' \times 6''$ timber, so as to bring their surfaces flush with that of the rafter and provide a practical construction.

The greatest stress in the wooden tie-beam is that in KZ, 28,600 pounds; from Table 6, Art. **61,** the ultimate tensile strength of yellow pine is 8,000 pounds per square inch, which, with a factor of safety of 5, furnishes an allowable unit stress of $8{,}000 \div 5 = 1{,}600$ lb. per sq. in. The net sectional area must, then, be $28{,}600 \div 1{,}600 = 18$ sq. in., nearly; if we assume that the net area is two-thirds of total area, the timber must have a total sectional area of $18 \times \frac{3}{2} = 27$ sq. in.; the nearest even size that will fill the conditions is a $6'' \times 6''$ timber.

The stress in the tie-rod PQ is 11,400 pounds; using the value for wrought iron given in Table 6, the allowable tensile strength of the rod is $50{,}000 \div 5 = 10{,}000$ lb. per sq. in., and its net area must be $11{,}400 \div 10{,}000 = 1.14$ sq. in.; the area at the root of the thread of a $1\frac{1}{2}$-inch bolt is 1.28 sq. in., and this is the smallest size that will provide the area demanded.

For the rod NO a net area of $3{,}800 \div 10{,}000 = .38$ sq. in.

is demanded, which will require a diameter of $\frac{7}{8}$ in. The net area of ML is to be not less than $1{,}900 \div 10{,}000 = .19$ sq. in., and since area at the root of the thread of a $\frac{5}{8}$-inch bolt is .196 sq. in., this size may, therefore, be used with safety.

In order to provide sufficient strength at the foot of the rafter to prevent shearing the end of the tie, it will be necessary to use some device similar to the wrought-iron strap shown in Fig. 99, Art. **133,** as will be readily seen from the following: Assuming that the tie projects 24 inches beyond the end of the rafter, the area of wood which, by its shearing strength, acts to resist the pull of the tie, is $6 \times 24 = 144$ sq. in.; from Table 6, the ultimate shearing strength of yellow pine parallel to the grain is 400 pounds per square inch; with a factor of safety of 5, this provides an allowable stress of $400 \div 5 = 80$ lb. per sq. in., making the total allowable stress on the section $144 \times 80 = 11{,}520$ pounds, which leaves $28{,}600 - 11{,}520 = 17{,}080$ pounds to be provided for by the use of some extra fastening.

The details of the truss may be similar to those presented in Fig. 99, and, by referring to Art. **133,** the student should have no difficulty in making a drawing with all needed details for the truss under consideration.

(71) See Arts. **44** to **46.**

(72) Since the truss is symmetrical and symmetrically loaded, but one-half of the stress diagram is required. Fig. 15 is the frame diagram, and Fig. 16 the stress diagram.

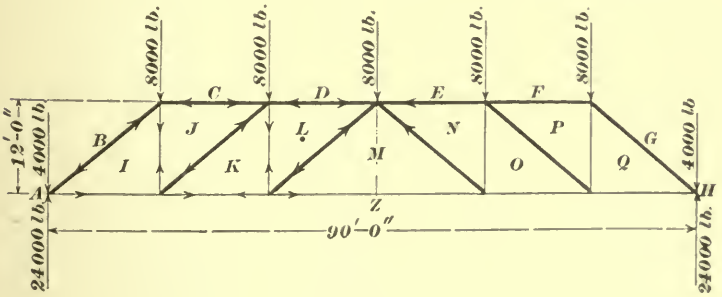

FIG. 15.

The construction of the latter should be perfectly obvious from the principles and examples presented in Arts. **119** to **132.**

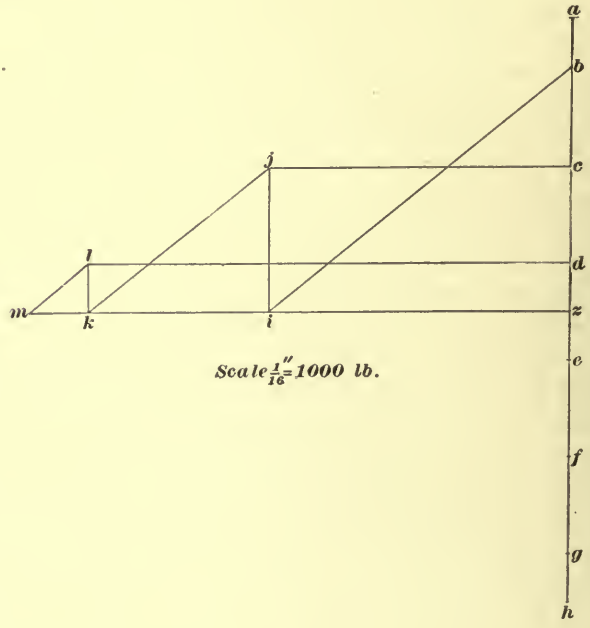

FIG. 16.

The stresses in the several numbers composing the left-hand half of the truss are as follows:

Members.	Stress. Lb.	Kind of Stress.
B I	32,000	Compression.
C J	25,000	Compression.
D L	40,000	Compression.
I J	12,000	Tension.
J K	19,200	Compression.
K L	4,000	Tension.
L M	6,400	Compression.
I Z	25,000	Tension.
K Z	40,000	Tension.
M Z	45,000	Tension.

Architectural Engineering.

(QUESTIONS 1–50. SEC. 6.)

(1) (*a*) From the table "Elements of Usual Sections," the formula for the moment of inertia of a solid circle with respect to an axis through its center is $I = \frac{A\,D^2}{16}$; the area of a 4-inch circle is $4^2 \times .7854 = 12.566$ sq. in. Substituting, in the formula, we have

$$I = \frac{12.566 \times 4^2}{16} = 12.566. \quad \text{Ans.}$$

(*b*) Substituting, in formula **1**, Art. **7**, we have,

$$I' = I + a\,r^2 = 12.566 + 12.566 \times 8^2 = 816.79. \quad \text{Ans.}$$

(2) (*a*) From the table "Elements of Usual Sections," the formula for the moment of inertia of a solid rectangle is $I = \frac{b\,h^3}{12}$; the moment of inertia of the web-plate with respect to the axis *a b*, is, therefore,

$$I' = I = \frac{\frac{1}{2} \times 12^3}{12} = 72.$$

From the table "Properties of Angles, Unequal Legs," the area of a $6'' \times 3\frac{1}{2}'' \times \frac{3}{8}''$ angle is 3.42 sq. in.; the distance of its center of gravity from the back of the longer flange is .79 inch, and its moment of inertia, with respect to an axis

through the center of gravity parallel to the longer flange, is 3.34. From the dimensions of the section, the distance of the center of gravity of the angle from the axis $a\,b$ is 6 in. $-$ .79 in. $=$ 5.21 in. Substituting now in formula **1**, Art. **7**, the moment of inertia of one of the angles with respect to the axis $a\,b$ is

$$I' = 3.34 + 3.42 \times 5.21^2 = 96.1728.$$

According to formula **2**, Art. **7**, the moment of inertia of the whole section with respect to the axis $a\,b$ is

$$I_s = \Sigma I' = 72 + 4 \times 96.1728 = 456.6912. \quad \text{Ans.}$$

(*b*) Applying the formula for the moment of inertia of a solid rectangle with respect to an axis through its center of gravity, the moment of inertia of the web-plate with respect to the axis $c\,d$ is

$$I' = I = \tfrac{1}{12} \times 12 \times (\tfrac{1}{2})^3 = .125.$$

From the table "Properties of Angles, Unequal Legs," the distance of the center of gravity of a $6'' \times 3\tfrac{1}{2}'' \times \tfrac{3}{8}''$ angle from the back of the shorter flange is 2.04 inches, and the moment of inertia of the angle, with respect to an axis through the center of gravity parallel to the shorter flange, is 12.86. The distance of the center of gravity of the angle from the axis $c\,d$, as shown by the section, is 2.04 + .25 $=$ 2.29 in.; the moment of inertia of the angle with respect to $c\,d$ is, therefore,

$$I' = 12.86 + 3.42 \times 2.29^2 = 30.7948.$$

The moment of inertia of the whole section with respect to the axis $c\,d$ is

$$I_s = \Sigma I' = .125 + 4 \times 30.7948 = 123.3042. \quad \text{Ans.}$$

(*c*) The total area of the section is $A = \tfrac{1}{2} \times 12 + 4 \times 3.42$ $=$ 19.68 sq. in., and its least moment of inertia is 123.3042; substituting these values in formula **6**, Art. **11**, the square of the least radius of gyration is

$$K^2 = \frac{123.3042}{19.68} = 6.26. \quad \text{Ans.}$$

(3) Applying formula **9,** Art. **15,** the ultimate compressive strength of a structural-steel column 18 feet long, the square of whose least radius of gyration is 6.26, is in round numbers,

$$S = \frac{52{,}000}{1 + \frac{216^2}{18{,}000 \times 6.26}} = 36{,}775 \text{ lb. per sq. in.};$$

with a factor of safety of 5, the allowable unit stress is $36{,}775 \div 5 = 7{,}355$ lb. per sq. in.; and, since the area of the section is 19.68 sq. in., the column will safely support a load of $19.68 \times 7{,}355 = 144{,}764$ lb., or in round numbers, 144,800 lb. Ans.

(4) The total uniformly distributed load is $1{,}500 \times 24 = 36{,}000$ lb., and the length of the beam is $24 \times 12 = 288$ inches; substituting these values in the formula $M = \frac{WL}{8}$ (see Table 10, *Architectural Engineering*, § 5, Art. **97**), the maximum bending moment on the beam is found to be

$$M = \frac{36{,}000 \times 288}{8} = 1{,}296{,}000 \text{ in.-lb.}$$

In the table "Properties of **I** Beams," the moment of inertia of a 15-inch 60-pound beam is found to be 637.7; the distance of its neutral axis from the most distant fiber is $7\frac{1}{2}$ inches. Substituting the known values in formula **4,** Art. **10,** we have $1{,}296{,}000 = S\frac{637.7}{7.5}$, from which

$$S = \frac{1{,}296{,}000 \times 7.5}{637.7} = 15{,}240 \text{ lb. per sq. in.} \quad \text{Ans.}$$

(5) (*a*) See Art. **18.**

(*b*) The cost of labor and the difficulty in making connections to the column with the best theoretical section may more than compensate for any saving in material by its use. See Art. **18.**

(6) The total load on the girder is $24 \times 18 \times 400 = 172{,}800$ lb.; since the load is uniformly distributed, the reaction at each support is one-half of this, or 86,400 pounds, which is

the load that must be supported by the rivets. From the table "Values of Rivets," the allowable stress on a $\frac{3}{4}$-inch rivet in ordinary bearing in a $\frac{5}{16}$-inch plate is 3,656 pounds; the number of rivets required is, therefore, $86,400 \div 3,656 = 24$, nearly. Ans.

(7) (*a*) 6 inches. See Art. **20.**

(*b*) In the table "Values of Rivets," the minimum distance of the center of a $\frac{3}{4}$-inch rivet from the edge of a $\frac{5}{16}$-inch plate is given as $1\frac{3}{16}$ inches; or, by applying the rule for distance of center of rivet from end of plate, as given in the table, we have, end distance $= \frac{5}{16} + \frac{1}{2} \times \frac{3}{4} + \frac{1}{2} = 1\frac{3}{16}$ in. Ans.

(8) (*a*) and (*b*) See Art. **66.**

(9) In accordance with the definition of the modulus of elasticity (see Art. **60**), the modulus is the quotient obtained by dividing the unit stress by the unit strain. The sectional area of a $\frac{1}{2}$-inch bar is $.7854 \times (\frac{1}{2})^2 = .19635$ sq. in.; the unit stress is, therefore, $\frac{7,000}{.19635}$ lb. per sq. in. Since a bar $7\frac{1}{2}$ inches long was elongated .009 inch, the unit strain was $\frac{.009}{7.5}$ inch. Dividing the unit stress by the unit strain, the modulus of elasticity is, in round numbers,

$$\frac{7,000}{.19635} \div \frac{.009}{7.5} = 29,709,000 \text{ lb. per sq. in.} \quad \text{Ans.}$$

(10) (*a*) To find the center of gravity of the figure we will apply the principle of moments, Art. **3.** Taking the upper edge as the axis of moments, the moments of the areas of the two rectangles are as follows:

	Area.	Lever Arm.	Moment.
Upper rectangle.	$8 \times 1\frac{1}{2} = 12$	$1\frac{1}{2} \div 2 = \frac{3}{4}$	9
Lower rectangle.	$8\frac{1}{2} \times 2 = 17$	$8\frac{1}{2} \div 2 + 1\frac{1}{2} = 5\frac{3}{4}$	97.75
Totals,	29		106.75

The distance of the center of gravity from the upper edge; that is, the lever arm of the area of the whole figure

with respect to the upper edge as an axis, is $106.75 \div 29 = 3.681$ in., nearly.

The moment of inertia of the upper rectangle with respect to an axis through its center of gravity and parallel to its upper edge is

$$I = \frac{b\,h^3}{12} = \frac{8 \times (1\frac{1}{2})^3}{12} = 2.25$$

(see table "Elements of Usual Sections"); the distance of the center of gravity of the upper rectangle from the axis through the center of gravity of the whole figure is $3.681 - .75 = 2.931$ in., and the area of the rectangle, as determined above, is 12 sq. in.; therefore, its moment of inertia, with respect to the required axis, is

$$I' = I + A\,r^2 = 2.25 + 12 \times 2.931^2 = 105.342.$$

See formula **1,** Art. **7.**

The moment of inertia of the lower rectangle, with respect to an axis through its center of gravity and parallel to the required axis, is

$$I = \frac{2 \times (8\frac{1}{2})^3}{12} = 102.354;$$

the distance of its center of gravity from the required axis is $5.75 - 3.681 = 2.069$ in., and its area is 17 sq. in. Therefore, the moment of inertia, with respect to the required axis, is

$$I' = 102.354 + 17 \times 2.069^2 = 175.127.$$

By applying formula **2,** Art. **7,** the moment of inertia of the whole figure, with respect to the required axis, is

$$I_s = \Sigma I' = 105.342 + 175.127 = 280.469. \quad \text{Ans.}$$

(*b*) In order to calculate the least radius of gyration we must know the least moment of inertia. The moment of inertia of the figure with respect to the axis through its center of gravity and parallel to the upper flange being known, we will now calculate it for the axis perpendicular to this flange. The moment of inertia of the upper rectangle, with respect to this axis, is

$$I = \frac{1\frac{1}{2} \times 8^3}{12} = 64,$$

and the moment of inertia of the lower rectangle, with respect to the same axis, is

$$I = \frac{8\frac{1}{2} \times 2^3}{12} = 5.667;$$

the moment of inertia of the whole section is, therefore,

$$I_s = \Sigma I = 64 + 5.667 = 69.667.$$

Since this is less than the value found for the moment, with respect to the axis parallel to the upper edge, it is the value to be used in calculating the least radius of gyration; applying formula **5,** Art. **11,** we have

$$R = \sqrt{\frac{I}{A}} = \sqrt{\frac{69.667}{29}} = 1.55. \quad \text{Ans.}$$

(11) (*a*) and (*b*) See Art. **34.**

(12) (*a*), (*b*), and (*c*) See Art. **35.**

(13) The amount to be deducted from the width of the plate for each rivet hole is $\frac{3}{4} + \frac{1}{8} = \frac{7}{8}$ in.; the net width is, therefore, $30 - 9 \times \frac{7}{8} = 22$ in., nearly; applying formula **12,** Art. **38,** the required thickness of the plate is

$$T = \frac{86{,}400}{22 \times 12{,}000} = .327 \text{ in.}$$

The next regular thickness above this is $\frac{3}{8}$ inch, which is the thickness that should be used. Ans.

(14) From the table "Areas of Angles," the area of a $6'' \times 4'' \times \frac{1}{2}''$ angle is found to be 4.75 square inches; in accordance with the statement of the problem, the section to be deducted from this is $2 \times \frac{7}{8} \times \frac{1}{2} = \frac{7}{8} = .875$ sq. in., leaving a net area of $4.75 - .875 = 3.875$ sq. in. for each angle. The area of one flange plate is $8 \times \frac{3}{8} = 3$ in., and the section to be deducted, $2 \times \frac{7}{8} \times \frac{3}{8} = .656$ sq. in., leaving a net area of $3 - .656 = 2.344$ sq. in. The total net area is:

2 angles, each 3.875 sq. in.	7.7 5 0 sq. in.
4 cover-plates, each 2.344 sq. in.	9.3 7 6 sq. in.
Total,	1 7.1 2 6 sq. in.

Applying formula **15**, Art. **44**, for the outer plate, we have

$$l = 75\sqrt{\frac{2.344}{17.126}} = 27.75 \text{ ft.} = 27 \text{ ft. } 9 \text{ in.} \quad \text{Ans.}$$

For the second plate the area of the plates to and including the plate in question is $2 \times 2.344 = 4.688$, and the required length is

$$l = 75\sqrt{\frac{4.688}{17.126}} = 39.24 \text{ ft.} = 39 \text{ ft. } 3 \text{ in.} \quad \text{Ans.}$$

For the third plate the area a is $3 \times 2.344 = 7.032$ sq. in., and the length

$$l = 75\sqrt{\frac{7.032}{17.126}} = 48.06 \text{ ft., say } 48 \text{ ft. } 1 \text{ in.} \quad \text{Ans.}$$

For the fourth plate we have $a = 4 \times 2.344 = 9.376$ sq. in., and

$$l = 75\sqrt{\frac{9.376}{17.126}} = 55.49 \text{ ft.} = 55 \text{ ft. } 6 \text{ in.} \quad \text{Ans.}$$

(15) (*a*) See Art. **59.**
(*b*) See Art. **57.**
(*c*) See Art. **58.**

(16) The total load on the beam is $2 \times 14 \times 150 = 4{,}200$ lb., and its length is $14 \times 12 = 168$ in. From the table "Properties of I Beams," the moment of inertia of a 10-inch 25-pound beam is 122.5, and from the table "Modulus of Elasticity of Metals," the modulus of structural steel is found to be 29,000,000. Substituting the given values in the formula for the deflection of beam with both ends supported and a uniformly distributed load, the deflection is

$$d = \frac{5 \times 4{,}200 \times 168^3}{384 \times 29{,}000{,}000 \times 122.5} = .073 \text{ in.} \quad \text{Ans.}$$

(17) The total length of the rafter being 52 feet, the length of one panel is $52 \div 4 = 13$ ft.; the total uniformly distributed load on a panel is $13 \times 600 = 7{,}800$ lb.; the bending moment is, therefore,

$$M = \frac{7{,}800 \times 156}{8} = 152{,}100 \text{ in.-lb.}$$

In Table 6, Art. **61,** *Architectural Engineering*, § 5, the modulus of rupture of yellow pine is given as 7,300 pounds per square inch; with a factor of safety of 4 this gives an allowable stress of $7{,}300 \div 4 = 1{,}825$ lb. per sq. in., and the section required to resist the bending moment must have a modulus of

$$K = \frac{152100}{1825} = 83.34$$

(see formula **4,** Art. **10,** also Art. **70**). Since the depth of the beam is 10 inches, the breadth is found, by applying formula **15,** Art. **101,** *Architectural Engineering*, § 5, to be

$$b = \frac{6K}{d^2} = \frac{6 \times 83.34}{10^2} = 5 \text{ in.}$$

This is the thickness of the rafter that is required to resist the bending stress.

Since the section of the rafter is relatively short, its compressive strength may be considered as that of a short column. In Table 6, Art. **61,** *Architectural Engineering*, § 5, the compressive strength of yellow pine is stated to be 4,400 pounds per square inch; with a factor of safety of 4, the allowable stress is $4{,}400 \div 4 = 1{,}100$ lb. per sq. in., and the area required to resist the compressive stress is $56{,}000 \div 1{,}100 = 50.9$ sq. in.; the depth being 10 in., the thickness must be $50.9 \div 10 = 5.09$ in. The total thickness of the rafter must, therefore, be $5 + 5.09 = 10.09$, say 10 in. Ans.

(18) Applying the formula $I = \frac{bd^3}{12}$, see table "Elements of Usual Sections," the moment of inertia of the section of the beam about its neutral axis when the longer side is vertical is

$$I = \frac{8 \times 14^3}{12} = 1{,}829\tfrac{1}{3},$$

and when the shorter side is vertical the moment is

$$I = \frac{14 \times 8^3}{12} = 597\tfrac{1}{3}.$$

The total uniformly distributed load on the beam is $50 \times 16 = 800$ lb., and from the table "Modulus of Elasticity

of Timber," the modulus of white pine is found to be 1,000,000; applying the formula for the deflection of a simple beam uniformly loaded, see table "Deflection of Beams," the deflection, where the longer side is vertical, is

$$d = \frac{5 \times 800 \times 192^3}{384 \times 1{,}000{,}000 \times 1{,}829\frac{1}{3}} = .0403 \text{ in.};$$

when the shorter side is vertical, the deflection is

$$d = \frac{5 \times 800 \times 192^3}{384 \times 1{,}000{,}000 \times 597\frac{1}{3}} = .1234 \text{ in.}$$

The difference in deflection is, therefore, $.1234 - .0403 = .0831$ in. Ans.

(19) (*a*) The Fink truss. See Art. **69.**

(*b*) At one of the joints there are three unknown stresses to be determined, and such a determination is an impossibility with the usual method of the polygon of forces. See Art. **67.**

(*c*) Owing to the construction of the truss, it can be proved that one of the unknown stresses is equal to the stress in a member that has been previously determined; the unknown stresses are thus reduced to two, which makes it possible to complete the force polygon. See Art. **67.**

(20) The necessity for providing suitable connections and the durability of the member when exposed to the corrosive action of moisture and gases make it important that the section of metal should never be less than a certain quantity. See Art. **73.**

(21) (*a*), (*b*), and (*c*) See Art. **64.**

(22) See Art. **44.**

(23) See Arts. **51** to **54.**

(24) Taking the center of moments over the reaction R_1, the moments of the loads are as follows:

Load *a*,	8,4 0 0×10 =	8 4,0 0 0 ft.-lb.
Load *b*,	2 0,6 0 0×25 =	5 1 5,0 0 0 ft.-lb.
Load *c*,	1 9,0 0 0×40 =	7 6 0,0 0 0 ft.-lb.
Load *d*,	2 2,8 0 0×58 =	1,3 2 2,4 0 0 ft.-lb.
Load *e*,	1 3,8 0 0×72 =	9 9 3,6 0 0 ft.-lb.
Totals,	8 4,6 0 0	3,6 7 5,0 0 0 ft.-lb.

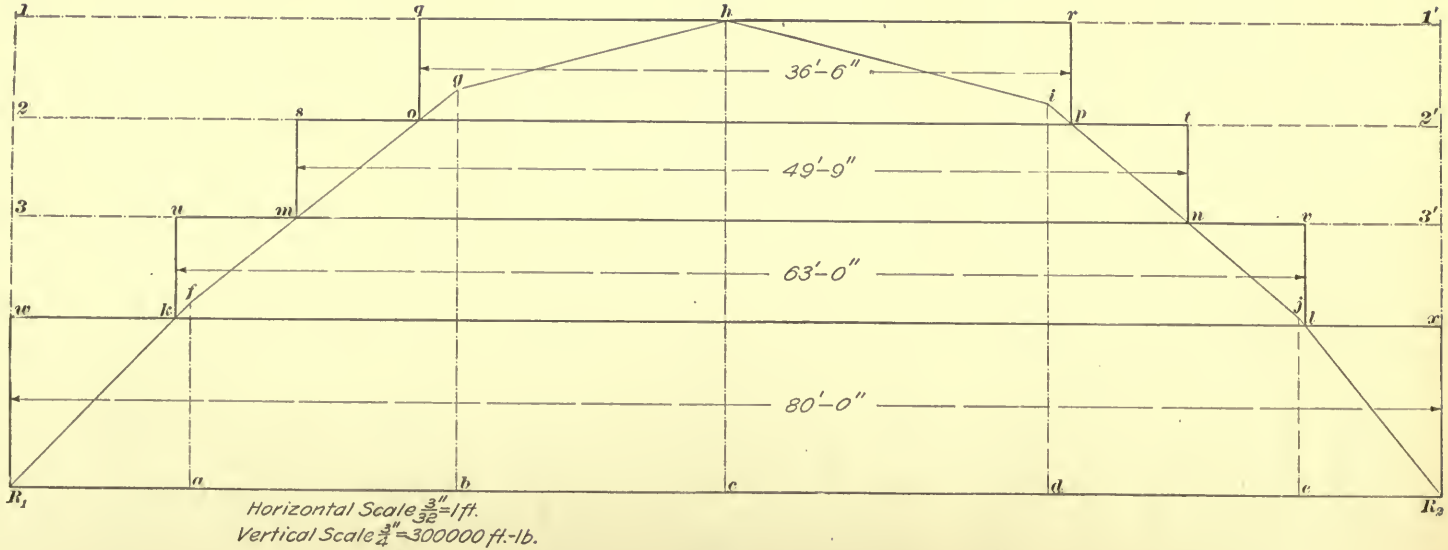
1
q
h
r
1'
36'-6"
g
2
s
o
i
p
t
2'
49'-9"
3
u
m
n
v
3'
63'-0"
w
k
f
j
l
x
80'-0"
a
b
c
d
e
R_1
R_2
Horizontal Scale $\frac{3}{32}$" = 1 ft.
Vertical Scale $\frac{3}{4}$" = 300000 ft.-lb.

FIG. 1.

The reaction R_2 is $3,675,000 \div 80 = 45,937.5$ lb., and at R_1 the reaction is, therefore, $84,600 - 45,937.5 = 38,662.5$ lb.

The bending moments under the various loads are as follows:

Under load a, $38,662.5 \times 10 = 386,625$ ft.-lb.;
Under load b, $38,662.5 \times 25 - 8,400 \times 15 = 840,562.5$ ft.-lb.;
Under load c, $38,662.5 \times 40 - (8,400 \times 30 + 20,600 \times 15)$
$= 985,500$ ft.-lb.;
Under load d, $45,937.5 \times 22 - 13,800 \times 14 = 817,425$ ft.-lb.;
Under load e, $45,937.5 \times 8 = 367,500$ ft.-lb.

The net area of one of the flange plates is $10 \times \frac{1}{2} - 2 \times \frac{7}{8} \times \frac{1}{2} = 4.125$ sq. in. From the table "Areas of Angles," the area of a $4'' \times 4'' \times \frac{5}{8}''$ angle is 4.61 sq. in.; the total section to be deducted from the two angles for the rivet holes is $4 \times \frac{7}{8} \times \frac{5}{8} = 2.19$ sq. in.; the net area of the two angles is, therefore, $2 \times 4.61 - 2.19 = 7.03$ sq. in., and the net area of the whole section is $3 \times 4.125 + 7.03 = 19.405$ sq. in.

Now construct the diagram as described in Art. **47**; draw the line $R_1 R_2$, Fig. 1, making its length correspond to that of the beam, and on it locate the points of application of the loads a, b, c, d, and e. From the ends R_1 and R_2 and the points of application of the loads, draw indefinite perpendiculars to $R_1 R_2$. Choosing a vertical scale of bending moments such that a height of 1 inch is equal to a bending moment of 400,000 ft.-lb., the bending moments at the points of application of the loads are, respectively, represented by the following vertical distances:

At a, $386,625 \div 400,000 = .967$ in.
At b, $840,562.5 \div 400,000 = 2.10$ in.
At c, $985,500 \div 400,000 = 2.46$ in.
At d, $817,425 \div 400,000 = 2.04$ in.
At e, $367,500 \div 400,000 = .92$ in.

These distances laid off on the verticals from the points a, b, c, etc., locate the points f, g, h, i, and j, which fix the polygon $R_1 f g h \ldots R_2$.

Through the highest point h of the polygon, draw the horizontal line 1–$1'$; then divide the perpendicular distance $1 R_1$ into parts proportional to the sectional areas of the

flange plates and the sum of the areas of the two flange angles. Through the points of division *2*, *3*, and *w*, draw the horizontal lines *2–2′*, *3–3′*, and *w x*; at the points *k*, *m*, *o*, *p*, *u*, and *l*, where these lines cut the polygon $R_1 f g \ldots R_2$, erect the perpendiculars *k u*, *m s*, *o q*, etc., thus forming the rectangles *k u v l*, *m s t n*, and *o q r p*. The lengths of these rectangles, when measured by the scale to which the load line $R_1 R_2$ was laid off ($\frac{3}{32}$ in. = 1 foot in the present example), give the required lengths of the cover-plates, 36 ft. 6 in., 49 ft. 9 in., and 63 ft. 0 in., respectively. Ans.

(25) (*a*) Proportioning the stiffness to withstand the greater reaction, the minimum sectional area required is $45{,}937.5 \div 12{,}000 = 3.83$ sq. in. for the two angles, or $3.83 \div 2 = 1.915$ sq. in. for each; by reference to the table "Areas of Angles," it is seen that the area of a $2\frac{3}{4}'' \times 2\frac{3}{4}'' \times \frac{3}{8}''$ angle is 1.92 sq. in., consequently, this size of angle may be used.

(*b*) The net depth of the web-plate is $60 - 15 \times \frac{7}{8} = 46\frac{7}{8}$ in. Applying formula **12**, Art. **38**, the calculated thickness is

$$T = \frac{45{,}937.5}{46.875 \times 12{,}000} = .082 \text{ in.} = \tfrac{3}{32} \text{ in., nearly.} \quad \text{Ans.}$$

(26) From the table "Areas of Angles," the total area of a $6'' \times 6'' \times \frac{7}{16}''$ angle is 5.06 sq. in.; the diameter of the rivet hole is $\frac{7}{8} + \frac{1}{8} = 1$ in.; therefore, the section to be deducted for a $\frac{7}{8}$-inch rivet is $1 \times \frac{7}{16} = \frac{7}{16}$ sq. in., and the net area is $5.06 - 2 \times \frac{7}{16} = 4.185$ sq. in. Ans.

(27) (*a*) $\frac{1}{15}$ of the span. See Art. **37.**

(*b*) The girder should be so proportioned that the deflection will not be excessive. See Art. **37.**

(28) (*a*) At joints and at the foot of a built-up column the rivets are usually spaced closer than in the body of the column. See Art. **20.**

(*b*) Where the aggregate thickness of the metal between the heads of the rivet is not more than $1\frac{1}{4}$ inches, a $\frac{3}{4}$-inch rivet may be used; if the aggregate thickness of the several

plates is more than $1\frac{1}{4}$ inches and less than 3 inches, use $\frac{7}{8}$-inch rivets; and if the aggregate thickness is 3 inches or more, use 1-inch rivets. See Art. **20.**

(29) The inaccessibility of its inner surface for painting and inspection, and the difficulty of making connections for floorbeams, etc. See Art. **18.**

(30) From the table "Properties of Angles, Equal Legs," the distance of the center of gravity of a $3\frac{1}{2}'' \times 3\frac{1}{2}'' \times \frac{3}{8}''$ angle from the back of a flange is found to be 1.01 inches, its moment of inertia, with respect to an axis through this center and parallel to the back of the flange, is 2.87, and the area of the section is 2.48 square inches. The distance of the center of gravity of one of the angles from an axis through the center of gravity of the section and parallel to one of its sides is $6 - 1.01 = 4.99$ inches; the moment of inertia of an angle, with respect to this axis, by applying formula **1,** Art. **7,** is

$$I' = 2.87 + 2.48 \times 4.99^2 = 64.62;$$

and, from formula **2,** Art. **7,** the moment of inertia of the four angles with respect to the axis is

$$I_s = \Sigma I' = 4 \times 64.62 = 258.48,$$

which, since the effect of the lattice bars is not considered in calculating the moment of inertia, is the total moment of inertia of the section with respect to an axis through its center of gravity, parallel to either side

Applying formula **6,** Art. **11,** the square of the radius of gyration is

$$R^2 = \frac{258.48}{4 \times 2\ 48} = 26.06.$$

From Table 6, Art. **61,** *Architectural Engineering,* § 5, the ultimate compressive strength of structural steel is 52,000 pounds per square inch; the length of the column is $24 \times 12 = 288$ in.; substituting in formula **9,** Art. **15,** the ultimate compressive strength of the column is

$$S = \frac{52{,}000}{1 + \frac{288^2}{36{,}000 \times 26.06}} = 47{,}800 \text{ lb. per sq. in. of section, nearly.}$$

With a factor of safety of 4 the allowable compressive strength is $47,800 \div 4 = 11,950$ lb. per sq. in., and the allowable load on the column is $11,950 \times 9.92 = 118,500$ lb., in round numbers. Ans.

(31) (*a*) Angles and plates.

(*b*) The angles are placed back to back in pairs with the plates between them at the joints, and all are held together by rivets through the angles and plates. See Art. **76.**

(32) (*a*) By upsetting the ends, the section at the root of the threaded portion of the rod is made equal to the section of the main part of the rod; a considerable saving of material is thus effected.

(*b*) No, because the saving in material would be too small to pay for the extra labor required. See Art. **71.**

(33) The diameter of the pin should be from $\frac{3}{4}$ to $\frac{4}{5}$ of the width of the bar in flats, and $1\frac{1}{4}$ times the diameter of the bar in rounds. The sectional area of the metal around the eye should be 50 per cent. in excess of that of the rod or bar. See Art. **89.**

(34) The stresses in the roof, owing to the comparatively steady nature of the loading, can be calculated with greater certainty than in a bridge; also, owing to the same cause, the stresses are not so severe on the material of the roof. See Art. **84.**

(35) See Art. **31.** First find the horizontal and vertical components of the stresses in the members *c* and *d*, by means of the diagram shown in Fig. 2. In this diagram the lines *on* and *op* are drawn parallel, respectively, to the members *c* and *d*; the distance *on* is laid off, to a scale of 1 inch = 4,000 pounds, equal to 15,000 pounds, the stress in *c*; and *op*, to the same scale, is made equal to the stress of 13,400 pounds in the member *d*; the directions in which these stresses act are shown by the arrows. Resolving each of these stresses

into its horizontal and vertical components by drawing the lines $o\,m$ and $n\,m$, $o\,r$ and $p\,r$, the horizontal component of the stress in c is found to be 9,000 pounds and the vertical

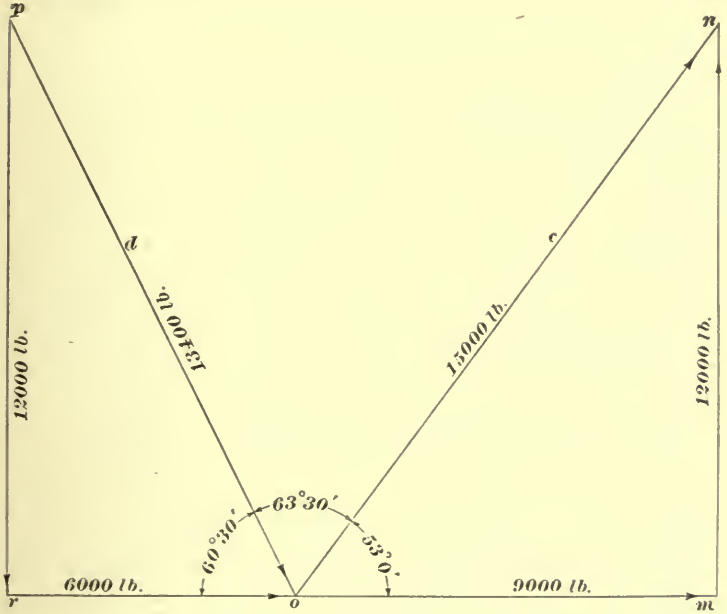

FIG. 2.

component 12,000 pounds; the respective stresses in d are 6,000 pounds horizontal, and 12,000 pounds vertical.

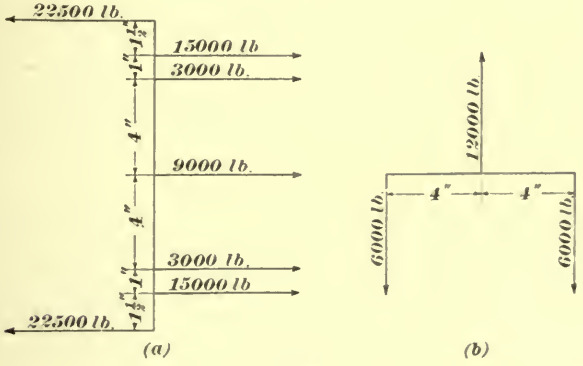

FIG. 3.

(*a*) Lay out the diagram of horizontal forces shown in Fig. 3 (*a*), and from it compute the greatest horizontal bending moment. By an inspection of this diagram, it is evident the greatest bending moment is at the center of the pin, where the load of 9,000 pounds is applied; its magnitude is given by the following calculation:

Taking the center of moments at the center of the pin, the moment of the force of 22,500 pounds is

$$22{,}500 \times 6\tfrac{1}{2} = \qquad 146{,}250 \text{ in.-lb.}$$

and the moments of the opposing forces are

$$15{,}000 \times 5 = 75{,}000 \text{ in.-lb.}$$
$$3{,}000 \times 4 = 12{,}000 \text{ in.-lb.} \qquad 87{,}000 \text{ in.-lb.}$$
$$\text{Remainder,} \qquad 59{,}250 \text{ in.-lb.}$$

the greatest horizontal bending moment on the pin. Ans.

(*b*) From the diagram of vertical forces, Fig. 3 (*b*), the vertical bending moment is $6{,}000 \times 4 = 24{,}000$ in.-lb. Ans.

(*c*) Since the maximum bending moment is equal to the resultant of the horizontal and vertical moments, its value is equal to the hypotenuse of a right triangle of which the vertical and horizontal stresses are the sides; applying this principle, the maximum bending moment is

$$\sqrt{59{,}250^2 + 24{,}000^2} = 63{,}926 \text{ in.-lb.} \quad \text{Ans.}$$

(*d*) From the table "Resisting Moments of Pins," we find that, with a unit fiber stress of 20,000 pounds, the resisting moment of a $3\frac{1}{4}$-inch pin is 67,400 inch-pounds, and that this is the smallest regular size that will fulfil the conditions of this problem; a $3\frac{1}{4}$-inch pin must, therefore, be used. Ans.

(36) (*a*) See Art. **81.** From the table "Values of Rivets," the allowable stress on a $\frac{3}{4}$-inch rivet in web bearing, with a $\frac{7}{16}$-inch plate and a unit tensile stress of 12,000 pounds per square inch, is 6,825 pounds. Using this value, the number of rivets required for each member is as follows:

Member a, $60{,}000 \div 6{,}825 = 9$, nearly.
Member b, $17{,}900 \div 6{,}825 = 3$, nearly.
Member c, $20{,}000 \div 6{,}825 = 3$, nearly.
Member d, $40{,}000 \div 6{,}825 = 6$, nearly.
} Ans.

(b) The number of rivets required for the horizontal members a and d makes it desirable to use a splice plate to connect these two members, rather than to use so many rivets through the gusset plate. If we assume 5 rivets through the gusset plate for the member a, there will remain a stress of $60{,}000 - 5 \times 6{,}825 = 25{,}875$ lb. yet to be provided for. With an allowable tensile stress of 12,000 pounds per square inch, the net area of the splice plate must be $25{,}875 \div 12{,}000 = 2.156$ sq. in.; using a plate $7\frac{1}{2}$ inches wide and $\frac{7}{16}$ inch thick, the net area, after deducting for two

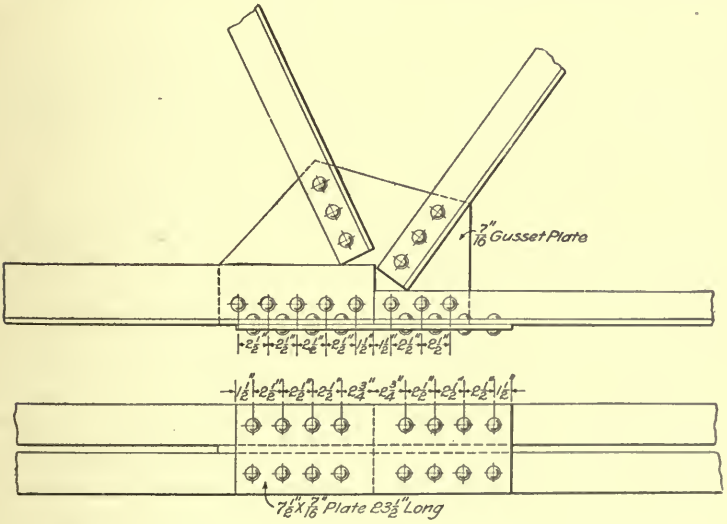

FIG. 4.

rivet holes, is $7\frac{1}{2} \times \frac{7}{16} - 2 \times \frac{7}{8} \times \frac{7}{16} = 2.51$ sq. in. Using $\frac{3}{4}$-inch rivets through the splice plate and horizontal legs of the angles, these rivets will be in single shear, and, since the thickness of the plates is greater than the greatest thickness of plate given in the table for the bearing value of $\frac{3}{4}$-inch

rivets, it will be necessary to calculate the rivets for shear. According to the rule for the shearing strength of rivets given in the table, the shearing strength of a $\frac{3}{4}$-inch rivet, when the allowable tensile stress is 12,000 lb. per sq. in., is $12{,}000 \times \frac{13}{15} \times \frac{5}{6} \times .442 = 3{,}831$ lb. per sq. in.; there will then be required $25{,}875 \div 3{,}831 = 7$ rivets, nearly, between each of the members a and d and the splice plate. For the purpose of making the construction symmetrical, we will use 8 rivets in each case, 4 through the leg of each angle; the construction of the joint, as now calculated, is shown in Fig. 4.

(37) From the table "Elements of Usual Sections," the moment of inertia of a hollow cylinder, with respect to an axis through the center, is

$$I = \frac{A D^2 - a d^2}{16};$$

the area A of the whole cylinder is $.7854 \times 12^2 = 113.10$, and the area a of the hollow portion is $.7854 \times 10^2 = 78.54$; substituting in the formula, we have

$$I = \frac{113.1 \times 12^2 - 78.54 \times 10^2}{16} = 527.025.$$

Substituting now in formula **1**, Art. **7**, the moment of inertia, with respect to the axis 18 inches from the center of the cylinder, is

$$I' = 527.025 + (113.1 - 78.54)\,18^2 = 11{,}724.465. \quad \text{Ans.}$$

(38) From the table "Radii of Gyration for Two Angles Placed Back to Back with Long Leg Vertical," the least radius of gyration of two $5'' \times 3'' \times \frac{1}{2}''$ angles, placed as stated in the question, is 1.42; the length of the column is $22 \times 12 = 264$ in., and the ultimate compressive strength of structural steel, from Table 6, Art. **61**, *Architectural Engineering*, § 5, is 52,000 lb. per sq. in.; substituting in formula **9**, Art. **15**, the ultimate compressive strength of the column is

$$S = \frac{52{,}000}{1 + \dfrac{264^2}{36{,}000 \times 1.42^2}} = 26{,}530 \text{ lb. per sq. in.}$$

With a factor of safety of 4, the allowable stress is $26,530 \div 4 = 6,632$ lb. per sq. in. From the table "Areas of Angles," the area of a $5'' \times 3'' \times \frac{3}{4}''$ angle is 5.44 sq. in.; the sectional area of the column is, therefore, $2 \times 5.44 = 10.88$ sq. in., and the allowable load is $10.88 \times 6,632 = 72,156$ lb., or in round numbers, 72,000 lb. Ans.

(39) The net depth of the web-plate is $48 - 14 \times \frac{7}{8} = 35.75$ in.; substituting in formula **12**, Art. **38**, the required thickness is

$$T = \frac{156,000}{35.75 \times 13,000} = .336 \text{ in., say } \tfrac{3}{8} \text{ in.} \quad \text{Ans.}$$

(40) The total load on the girder is $3,000 \times 84 = 252,000$ lb., and since this load is uniformly distributed, the reactions are each $252,000 \div 2 = 126,000$ lb. The length of each panel into which the web-plate is divided is $84 \div 14 = 6$ ft.; the decrease in vertical shear from panel to panel, working along the beam from either reaction towards the center, is, therefore, $6 \times 3,000 = 18,000$ lb. The vertical shear at either end of the girder is equal to the reaction, 126,000 pounds; according to the table "Values of Rivets," the web-bearing value of a $\frac{3}{4}$-inch rivet in a $\frac{5}{16}$-inch plate, when the allowable unit tensile stress is 15,000 pounds per square inch, is 6,094 pounds; therefore, according to the method described in Art. **54**, the number of rivets required in the first panel is $126,000 \div 6,094 = 21$, and the spacing for this panel is $72 \div 21 = 3.429$ in. At the beginning of the second panel the vertical shear is $126,000 - 18,000 = 108,000$ lb.; the number of rivets required is $108,000 \div 6,094 = 18$; and the spacing for the second panel, $72 \div 18 = 4$ in. At the beginning of the third panel the shear is $108,000 - 18,000 = 90,000$ lb.; the number of rivets required is $90,000 \div 6,094 = 15$; and the spacing for the third panel, $72 \div 15 = 4.8$ in. Similarly, for the fourth panel the shear is $90,000 - 18,000 = 72,000$ lb.; the number of rivets required, $72,000 \div 6,094 = 12$; and the spacing, $72 \div 12 = 6$ in. Since 6 inches is the maximum allowable spacing, it is unnecessary to carry the calculation any farther. Collecting the results obtained,

the spacings for the panels from the ends to the center are as follows: 1st panel, 3.429 in.; 2d panel, 4 in.; 3d panel, 4.8 inches; 4th, 5th, 6th, and 7th panels, each 6 inches. Ans.

(41) The total uniformly distributed load on a beam is $20 \times 2 \times 480 = 19{,}200$ lb.; the length of a beam, in inches, is $20 \times 12 = 240$ in.; the modulus of elasticity of structural steel, from the table "Modulus of Elasticity of Metals," is 29,000,000; and the moment of inertia of a 12-inch $31\frac{1}{2}$-pound **I** beam, from the table "Properties of **I** Beams," is 220.5. Substituting in the formula for the deflection of a simple beam uniformly loaded, see table "Deflection of Beams," the greatest deflection is

$$d = \frac{5 \times 19{,}200 \times 240^3}{384 \times 29{,}000{,}000 \times 220.5} = .54 \text{ in.} \quad \text{Ans.}$$

(42) The area of the section of the plate is $10 \times \frac{1}{4} = 2.5$ sq. in., and the distance of its center of gravity from the outer edge is $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$ in.; from the table "Properties of Angles, Unequal Legs," the area of a $6'' \times 3\frac{1}{2}'' \times \frac{7}{8}''$ angle is 7.55 square inches, and the distance of its center of gravity from the back of the shorter flange is 2.22 inches; the distance of its center of gravity from the outer edge of the plate is, therefore, $2.22 + .25 = 2.47$ in.

Applying, now, the principle of moments, see Arts. **3** and **4**, we have moment of plate, with respect to its outer edge as an axis, $2.5 \times \frac{1}{8} = .3125$. Moment of two angles, with respect to the outer edge of plate as an axis, $2 \times 7.55 \times 2.47 = 37.297$. Sum of the moments of the several sections, $37.297 + .3125 = 37.6095$; dividing by the sum of the areas, $37.6095 \div (2.5 + 2 \times 7.55) = 2.137$ in. Ans.

(43) The moment of inertia of the plate, with respect to an axis through its center of gravity and parallel to the axis $a\,b$, is

$$I = \frac{10 \times (\frac{1}{4})^3}{12} = .013$$

(see table "Elements of Usual Sections"); the distance of its neutral axis from the axis $a\,b$ is $2.137 - .125 = 2.012$ in.,

and its area is $10 \times \frac{1}{4} = 2.5$ sq. in. Applying formula **1**, Art. **7**, the moment of inertia of the plate, with respect to $a\,b$, is

$$I' = .013 + 2.5 \times 2.012^2 = 10.133.$$

From the table "Properties of Angles, Unequal Legs," the moment of a $6'' \times 3\frac{1}{2}'' \times \frac{7}{8}''$ angle, with respect to an axis through its center of gravity and parallel to the shorter flange, is 26.38; the distance of the center of gravity of the angle from the axis $a\,b$ is $2.47 - 2.137 = .333$ in. (see example 42); the moment of inertia of one of the angles, with respect to $a\,b$, is, therefore,

$$I' = 26.38 + 7.55 \times .333^2 = 27.217.$$

Applying formula **2**, Art. **7**, the moment of inertia of the whole section, with respect to $a\,b$, is

$$I_s = \Sigma I' = 10.133 + 2 \times 27.217 = 64.567. \quad \text{Ans.}$$

(44) (*a*) From the table "Properties of **I** Beams," the moment of inertia of 10-inch 33-pound beam, with respect to the given axis, is 161.3; substituting in formula **3**, Art. **9**, the section modulus is

$$K = \frac{161.3}{5} = 32.26. \quad \text{Ans.}$$

(*b*) Since, according to the table "Properties of Channels," the moment of inertia of a 12-inch 20-pound channel is 124.7, the section modulus is

$$K = \frac{124.7}{6} = 20.78. \quad \text{Ans.}$$

(45) Since the resisting moment is equal to the product of the section modulus multiplied by the greatest unit stress (see Art. **9**), (*a*) the resisting moment of the **I** beam is $32.26 \times 12{,}000 = 387{,}120$ in.-lb. Ans. (*b*) The resisting moment of the channel is $20.78 \times 12{,}000 = 249{,}360$ in.-lb. Ans.

(46) (*a*) The ability of the column to resist transverse or bending stresses decreases as its length increases, and this is provided for by the use of a greater factor of safety for the longer columns. See Art. **17**.

(*b*) See formula **11**, Art. **17**.

(47) See Art. **23.**

(48) (*a*) Either by the crippling of the plates or the shearing of the rivets. See Art. **26.**

(*b*) The compressive strength is assumed to be $\frac{13}{15}$ of the tensile strength and the shearing strength $\frac{5}{6}$ of the compressive strength. See Art. **27.**

(49) Bending and shearing. See Art. **29.**

(50) (*a*) One-sixth of the section of that part of the web-plate which is included between the flange angles is considered as a part of the effective flange area. See Art. **41.**

(*b*) The plate should not be spliced under the point of greatest bending moment and the splice should develop the greatest possible percentage of the strength of the plate. See Art. **41.**

MASONRY.

(QUESTIONS 1–79. SEC. 7.)

(1) (*a*) See Art. **28.**
(*b*) See Art. **30.**

(2) See Arts. **45** and **46.**

(3) (*a*) See Art. **92.**
(*b*) See Art. **94.**
(*c*) See Art. **92.**

(4) (*a*) See Art. **129.**
(*b*) See Art. **131.**
(*c*) See Art. **133.**

(5) See Arts. **144** to **148.**

(6) See Art. **169.**

(7) See Art. **200.**

(8) See Arts. **224** to **227.**

(9) See Art. **241.**

(10) See Art. **237.**

(11) See Arts. **276** to **279.**

(12) (*a*), (*b*), and (*c*) See Art. **182.**

(13) (*a*) and (*b*) See Art. **282.**
(*c*) See Arts. **282** and **284.**

(14) A trimmer arch. See Art. **234.**

(15) (*a*) and (*b*) See Art. **262.**
(*c*) See Art. **263.**

(16) See Art. **53.**

(17) (*a*) and (*b*) See Art. **99.**
(*c*) See Art. **100.**

(18) See Arts. **255** and **256.**

(19) See Arts. **203** to **213.**

(20) (*a*) See Art. **73.**
(*b*) See Art. **75.**
(*c*) See Art. **74.**
(*d*) $W = 1{,}200$ lb.; $h = 15$ ft. $= 180$ in.; and $a = \frac{5}{8}$ in.

Hence, $P = \dfrac{Wh}{8a} = \dfrac{1{,}200 \times 180}{8 \times \frac{5}{8}} = 43{,}200$ lb.

(*e*) See Art. **80.**

(21) See Art. **25.**

(22) See Art. **272.**

(23) (*a*) and (*b*) See Art. **247.**

(24) See Art. **254.**

(25) (*a*) See Art. **267.**
(*b*) See Art. **268.**

(26) (*a*) See Art. **194.**
(*b*) See Art. **196.**
(*c*) See Arts. **197** and **198.**

(27) See Art. **142.**

(28) See Art. **112.**

(29) See Arts. **266** and **267.**

(30) See Arts. **235** to **237.**

(31) (*a*) See Art. **275.**
(*b*) See Art. **274.**

(32) See Art. **240.**

(33) See Art. **243.**

(34) See Art. **238.**

(35) See Arts. **214** to **216.**

(36) See Art. **269.**

(37) See Table 4, Art. **280.**

(38) See Arts. **242, 243, 246, 248,** and **249.**

(39) See Art. **228.**

(40) See Art. **243.**

(41) See Art. **184.**

(42) See Arts. **219** to **223.**

(43) See Art. **20.**

(44) (*a*) See Art. **66.**
(*b*) See Art. **67.**
(*c*) See Art. **66.**

(45) See Art. **199.**

(46) See Arts. **229** and **230.**

(47) See Art. **97.**

(48) The load per sq. ft. on each floor is $75 + 100 = 175$ lb.; for 4 floors it is $175 \times 4 = 700$ lb. Adding the roof load, 20 lb. per sq. ft., the total is 720 lb. The area carried by each pier is 15 ft. $\times$ 15 ft. $= 225$ sq. ft. Multiplying 720 by 225, the total weight is 162,000 lb.; which, at 6,000 lb. per sq. ft., will require 27 sq. ft. of footing, or about 5.2 ft. square. Ans.

(49) See Art. **105.**

(50) See Arts. **244** and **245.**

(51) See Art. **228.**

(52) See Art. **265.**

(53) See Art. **266.**

(54) See Arts. **57** to **64.**

(55) (*a*) See Art. **163.**
(*b*) See Arts. **164** to **169.**

(56) See Arts. **113** and **114.**

(57) See Art. **82.**

(58) See Art. **86.**

(59) See Art. **188.**

(60) (*a*) See Art. **250.**
(*b*) See Art. **251.**

(61) See Art. **246.**

(62) (*a*) See Art. **158.**
(*b*) See Art. **160.**
(*c*) See Art. **158.**

(63) See Arts. **117** to **120.**

(64) See Arts. **124** to **128.**

(65) By means of cesspools. See Art. **31.**

(66) See Arts. **149** and **150.**

(67) By borings or trial pits. See Art. **7.**

(68) See Arts. **37** and **38.**

(69) See Arts. **88** and **89.**

(70) See Arts. **134** to **140.**

(71) (*a*) See Art. **190.**
(*b*) See Art. **192.**
(*c*) See Art. **193.**

(72) (*a*) Rock, virgin soil, and "made ground." See Art. **8.**
(*b*) Rock.
(*c*) See Table 1, Art. **19.**

(73) See Art. **271.**

(74) See Art. **217.**

(75) (*a*) and (*b*) .See Art. **191.**

(76) (*a*) See Arts. **175** to **178.**

(*b*) See Art. **172.**

(77) In the formula of Art. **122,** $b = 48$ in.; $d^3 = 6 \times 6 = 36$ in.; $l = 96$ in.; $A = .744$. The safe load is $\frac{1}{10}$ the breaking load, and the distributed load twice the concentrated or center load, hence, $\frac{2\,A\,b\,d^2}{10\,l}$ is the safe load. Substituting,

$$W = \frac{2 \times .744 \times 48 \times 36}{10 \times 96} = 2.68 \text{ tons.} \quad \text{Ans.}$$

(78) See Art. **162.**

(79) (*a*) and (*b*) See Arts. **152** to **156.**

MASONRY.

(QUESTIONS 1–78. SEC. 8.)

(1) (*a*), (*b*), and (*c*) See Art. **16.**
(*d*) See Art. **17.**

(2) (*a*) and (*b*) See Art. **54.**
(*c*) See Arts. **55** and **58.**

(3) (*a*) See Art. **154.**
(*b*) See Art. **155.**

(4) (*a*) See Art. **5.**
(*b*) See Art. **6.**
(*c*) See Art. **7.**

(5) (*a*) and (*b*) See Art. **231.**

(6) (*a*) and (*b*) See Art. **128.**

(7) (*a*) and (*b*) See Art. **216.**

(8) (*a*) See Art. **78.**
(*b*) and (*c*) See Art. **79.**

(9) (*a*) and (*b*) See Art. **232.**

(10) (*a*) See Art. **191.**
(*b*) See Art. **192.**

(11) In this example, $d = 9$ inches; $b = 6$ inches; $L = 5$ feet; $A = 70$; whence,

$$W = \frac{2\,b\,d^2\,A}{L} = \frac{2 \times 6 \times 9^2 \times 70}{5} = 13{,}608 \text{ lb.}$$

Safe load $= \frac{1}{6}\,W = \frac{13608}{6} = 2{,}268$ lb. Ans.

(12) (*a*) See Art. **234.**
(*b*) See Art. **238.**

(13) (*a*) See Arts. **137** and **138.**
(*b*) See Art. **145.**
(*c*) See Art. **148.**

(14) See Art. **70.**

(15) (*a*) See Art. **219.**
(*b*) See Art. **220.**

(16) (*a*) See Art. **88.**
(*b*) See Art. **90.**

(17) (*a*) See Art. **217.**
(*b*) and (*c*) See Art. **218.**

(18) (*a*) See Art. **261.**
(*b*) See Art. **262.**

(19) (*a*) and (*b*) See Art. **108.**

(20) (*a*) and (*b*) See Art. **104.**

(21) (*a*) See Art. **206.**
(*b*) See Art. **205.**
(*c*) See Art. **206.**

(22) (*a*) and (*b*) See Art. **199.**

(23) (*a*) See Art. **99.**
(*b*) See Art. **100.**

(24) (*a*) See Art. **223.**
(*b*) See Art. **224.**

(25) (*a*) See Art. **19.**
(*b*) See Art. **20.**

(26) (*a*) See Art. **204.**
(*b*) See Art. **205.**

(27) (*a*) See Art. **62.**
(*b*) See Art. **63.**
(*c*) See Art. **61.**

(28) (*a*) See Art. **166.**
(*b*) See Art. **170.**

(29) See Art. **249.**

(30) (*a*) See Art. **158.**
(*b*) See Art. **161.**

(31) (*a*) See Art. **245.**
(*b*) See Art. **246.**

(32) (*a*) See Art. **115.**
(*b*) See Art. **117.**
(*c*) See Art. **116.**

(33) (*a*) and (*b*) See Art. **185.**

(34)

	Highest.	Lowest.
Granite,	22,250	12,420
Marble,	20,000	7,610
Limestone,	25,000	6,650
Sandstone,	17,700	4,350

(35) (*a*) and (*b*) See Art. **14.**

(36) (*a*) See Arts. **186** and **187.**
(*b*) See Art. **188.**

(37) (*a*) and (*b*) See Art. **110.**

(38) (*a*) and (*b*) See Art. **93.**

(39) See Art. **118.**

(40) (*a*) See Arts. **23, 24,** and **25.**
(*b*) See Art. **24.**

(41) (*a*) See Art. **173.**
(*b*) See Art. **178.**

(42) (*a*) See Art. **97.**
(*b*) See Art. **98.**

(43) See Art. **134.**

(44) See Art. **243.**

(45) (*a*) See Art. **132.**
(*b*) See Art. **133.**

(46) (*a*), (*b*), and (*c*) See Art. **122.**

(47) (*a*) See Art. **244.**
(*b*) See Art. **242.**

(48) See Art. **256.**

(49) (*a*) See Art. **2.**
(*b*) See Arts. **3** and **4.**

(50) See Art. **250.**

(51) (*a*) See Arts. **194, 195,** also **252, 253.**
(*b*) See Art. **196.**

(52) See Art. **13.**

(53) See Art. **215.**

(54) (*a*) and (*b*) See Art. **226.**

(55) (*a*) and (*b*) See Art. **222.**

(56) See Art. **263.**

(57) (*a*), (*b*), and (*c*) See Art. **27.**
(*d*) See Art. **28.**

(58) (*a*) See Art. **123.**
(*b*) See Art. **124.**

(59) (*a*) and (*b*) See Art. **8.**

(60) (*a*) See Art. **49.**
(*b*) See Art. **53.**

(61) (*a*) See Art. **32.**
(*b*) In drove work the lines or grooves are broken, while in tooled work they are continuous across the surface.
(*c*) Worm-eaten.

(62) (*a*) See Art. **212.**
(*b*) See Art. **213.**

(63) (*a*) See Art. **113.**
(*b*) See Art. **114.**

(64) (*a*) See Art. **83.**
(*b*) and (*c*) See Art. **82.**

(65) (*a*) See Art. **44.**
(*b*) See Art. **47.**

(66) See Art. **182.**

(67) (*a*) and (*b*) See Art. **229.**

(68) (*a*) See Art. **210.**
(*b*) See Art. **212.**

(69) (*a*) See Arts. **75** and **76.**
(*b*) See Art. **74.**

(70) (*a*) and (*b*) See Art. **227.**

(71) (*a*) See Art. **130.**
(*b*) See Art. **129.**

(72) (*a*) See Art. **66.**
(*b*) See Art. **67.**

(73) (*a*) and (*b*) See Art. **208.**

(74) (*a*) and (*b*) See Art. **65.**

(75) See Art. **81.**

(76) (*a*) and (*b*) See Art. **207.**
(*c*) See Art. **209.**

(77) (*a*) See Art. **257.**
(*b*) See Art. **258.**

(78) (*a*) and (*b*) See Art. **221.**

CARPENTRY.

(QUESTIONS 1-80. SEC. 9.)

(1) The first course of shingles should be laid at the eaves and doubled. See Art. **157.**

(2) A partition should be trussed when there is no support for it from the story below, or when it is pierced by several doors or other openings, likely to render it insufficient to carry its superimposed load. See Art. **127.**

(3) Flashing on hips is applied in small pieces laid with each course of shingles, while flashing in the valleys of a roof is in the form of a continuous gutter left open between the slopes. See Arts. **162** and **163.**

(4) (*a*) A single floor consists of but one course, or layer, of boards nailed directly to the beams. See Art. **111.**

(*b*) A double floor consists of two courses of floor boards, the first being laid of rough material, and the second course, called the finished floor, being laid of better stock and blind nailed through the under floor to each beam.

(*c*) A single floor is always laid at right angles to the beams. A double floor is usually laid with its first or under course at an angle of about 45° with the beams, and its upper or finished floor at right angles to the beams. See Arts. **112** to **115.**

(5) (*a*) The water-table is a projecting strip around the lowest part of the woodwork of the building, on top of which the clapboards or siding finish.

(*b*) Its purpose is to receive the rain water that runs down the side of the house and throw it to the ground clear of the foundation wall. See Art. **188.**

(6) For first-class work, Georgia pine; for ordinary work, spruce; for cheap, rough framing, hemlock. See Arts. **42, 43,** and **45.**

(7) By boring a hole lengthwise through the column to ventilate the interior.

(8) (*a*) Three.

(*b*) Exogenous, endogenous, and acrogenous.

(*c*) The exogenous trees. See Arts. **8** and **9.**

(9) (*a*) The boards should be laid across the veranda from the house line to the edge, and with a pitch of $\frac{1}{4}$ inch to the foot to drain the water off.

(*b*) The joints should be laid in white lead. See Arts. **192** and **193.**

(10) When they are more than three feet wide. See Art. **129.**

(11) Because in cold climates the heat in the house is likely to cause the snow on the roof to melt and run down to the eaves, where, relieved from the influence of the internal heat, it is likely to freeze and form a dam, or ledge, of ice, and the water will then back up under the shingles and leak into the house. See Art. **166.**

(12) Matched flooring consists of planks which have been planed to a uniform thickness, and whose edges have been tongued and grooved to make a tight joint. See Art. **111.**

(13) A line is drawn representing the vertical axis of the tower, and to any given scale a section of a piece of siding is drawn at a distance from this axial line equal to the

external radius of the tower. The intersection of the prolonged outside line of the section of siding with the axial line of the tower, will then give the center from which the curve may be drawn, and the distance from this center to the upper and lower edges of the section of siding will be the radius of the upper and lower curves, respectively. See Art. **190.**

(14) Cedar, cypress, and chestnut.

(15) By means of iron dogs, which are bars with turned-over ends laid across the girder so that the turned ends fit into and tie together two floorbeams abutting on opposite sides of the girder. See Art. **221.**

(16) Duramen. See Art. **11.**

(17) One of the valley rafters is carried to the ridge of the main roof just as though both gables were of the same height, and the other valley is framed to the first one at the height of the lower ridge. See Art. **149.**

(18) Small windows projecting from the roof of a building. See Art. **167.**

(19) (*a*) Sixteen inches.

(*b*) So that the 4-foot lengths of lath will extend over four studs and get a bearing on each one without cutting or extending beyond. See Art. **116.**

(20) A chamfer consists of the surface of a cut angle or corner on a timber where such corner has been removed for a portion of the length only, and the timber retains its original shape for some distance from each end. See Art. **196.**

(21) To make a tight joint and keep the tops of the adjacent planks flush. See Art. **221.**

(22) Red cedar, redwood, and locust.

(23) The lines, radiating from the center of the tree toward the bark, which are seen in cross-sections of several of the hard woods. See Art. **15.**

(24) (*a*) A gable roof is one where the pitch falls in both directions from the center of the building.

(*b*) A gambrel roof is one with a gable whose slopes are broken so that the portion finishing at the eaves is steeper than the portion starting from the ridge. See Arts. **133** and **140.**

(25) (*a*) Centers are wood templets or molds, over which the voussoirs, or tapered stones of an arch, are placed to hold them in position until the mortar has set.

(*b*) They are constructed usually of two face pieces of $1\frac{1}{4}$-inch plank, which, after having been sawed to the proper curvature, are spaced the required distance apart and covered over with a number of transverse strips called *lagging.* See Arts. **189** to **201.**

(26) The sill is laid on the foundation one inch back from the outside edge of the wall, and is bedded in a layer of mortar which is pointed up after the sill is laid and leveled. See Art. **171.**

(27) (*a*) When a partition runs at right angles to the beams, the ends of the stud should stand on a sill laid across and spiked to the beams, unless there is a supporting partition immediately below, when the ends of the stud should stand on the cap of the supporting partition.

(*b*) When a partition runs parallel with the beams, it should stand on a beam framed in especially to receive it, or should rest on the cap of the partition below, if such a partition exists. See Arts. **117** and **120.**

(28) (*a*) A trimmer-arch center is one used over which to lay the bricks of a supporting arch for a fireplace hearth.

(*b*) It differs from the centers usually placed for templets for arched openings, as it is left in place even after the arch is complete and set, and the lath and plaster of the ceiling below are secured to it. See Art. **202.**

(29) Georgia pine.

(30) (*a*) A hip roof is one without gables where all four sides slope back at a uniform pitch from the eaves to an apex or ridge at the center.

(*b*) A Mansard roof is one where the sides pitch back in a broken slope, the portion at the eaves being nearly vertical, while the slope from the apex or ridge is nearly flat. See Arts. **135** and **141.**

(31) Georgia pine or spruce plank 3 inches thick and not more than 6 inches wide is best suited for the under floor in slow-burning construction, while $1\frac{1}{8}'' \times 4''$ maple or hard pine is suitable for the finished floor. See Arts. **218** and **221.**

(32) At the season when the sap has ceased to circulate, which in temperate climates is midsummer and midwinter, and in tropical climates, the dry season. See Art. **17.**

(33) A pendentive is the spherical triangle which brings the curved plan of a dome down to the angular corner or pier which supports it over the intersection of two passages. See Art. **213.**

(34) One-third. See Art. **76.**

(35) In order that the shrinkage may be equal and the settlement of the house uniform. See Art. **121.**

(36) When the angle of the room corresponds with an exterior angle of the building, the lath is secured against the corner stud for one wall, and against a nailing strip spiked to the corner stud for the other wall. When the angle of the room is against a straight exterior wall, the first stud of the interior partition is spiked to a 1-inch plank which is 2 inches wider than the stud and projects 1 inch beyond each side. The lath on the exterior wall are then secured to the projection of 1-inch plank and the lath on the interior wall are secured to the stud itself. See Arts. **118, 119,** and **122.**

(37) (*a*) A dome is the term applied to the roof of a building which is carried up either wholly or in part, in the form of a hemisphere or semiellipsoid.

(*b*) A dome is framed with rafters sawed to the required

curve, and framed between two circular members, called the sill and the plate, the former of which is equal to the plan of the base of the dome, and the latter to the plan of the opening or eye at the top.

(*c*) A dome may be covered with horizontal courses of planking, sawed to the required curve, and secured to the rafters; or it may be enclosed with vertical strips of plank, which are worked to a curved taper from their widest part at the base of the dome, to a pointed edge at the center of the top. These are secured to curved purlins framed in between the rafters at sufficient intervals to preserve the outline of the dome. See Arts. **203** to **206.**

(38) (*a*) It is a wood of the evergreen class, often substituted for Georgia pine in inferior work.

(*b*) No. See Art. **42.**

(39) Of 2-inch tongued-and-grooved plank laid vertically from floor to ceiling, which may be plastered over with wire lath on one side or lined with $\frac{7}{8}'' \times 3''$ matched boards of oak or Georgia pine. See Art. **273.**

(40) From 50 to 100 years. See Art. **18.**

(41) A groined ceiling is one formed over the intersection of two arched divisions or passages in a building. See Art. **209.**

(42) The interties are mortised into the corner posts so that the ones which run parallel to the floorbeams have their tops level with the tops of the floorbeams, and the ones which run at right angles with the floorbeams have their tops level with the bottom of the floorbeams which rest on them. See Art. **78.**

(43) To brace the structure on which it is laid. See Art. **180.**

(44) On a ledger board or girth let into the studs at such a height that the beams may be notched over it and spiked to each stud. See Art. **174.**

(45) For the construction of mills and factories, to obviate the difficulties met in the prevention or quenching of fires, so prevalent in structures of the braced-frame or balloon-frame construction. See Arts. **217** and **218.**

(46) (*a*) Spruce.

(*b*) Because it preserves well under water and resists the action of parasitic crustacea longer than any other similar wood. See Art. **43.**

(47) In braced-frame construction, each piece of the framework is carefully fitted and fastened to every other piece it comes in contact with, thus rendering the whole skeleton stiff and secure in itself, while in balloon-frame construction, the timbers and scantlings are simply nailed together lightly to hold them in place until the sheathing is applied to brace the structure and make it secure. See Art. **72.**

(48) They are provided with an air space around the ends in the wall by keeping the brickwork half an inch away from the beam or by using the Goetz-Mitchell or other suitable wall anchor. See Art. **226.**

(49) They are the circular cracks seen in the end sections of timber due to the separation of the fibers, from exposure to violent wind storms while the timber was standing. See Art. **22.**

(50) It is timber obtained from logs which have first been sawed in quarters, and each quarter then sawed into planks or scantling, as nearly as possible at right angles to the annual rings. See Art. **36.**

(51) (*a*) The square is set upon the rafter in such a position that the rise in inches is shown on the tongue, and the run in inches is indicated on the blade, both being on the same edge of the rafter. The tongue then marks the angle of the plumb-cut and the blade shows the bevel of the foot-cut, while the distance between the points where the tongue

and blade each cross the edge of the rafter will be $\frac{1}{12}$ the length of the rafter.

(*b*) The use of the fence obviates the necessity of a bevel in laying out the rafter cuts, or risers and treads of a stair string, etc., as by clamping it across the square from the tongue to the blade, these two elements of the square may be readily placed in their exact positions to mark the desired cuts. See Arts. **153** and **154.**

(52) In one method the studs are spaced regularly every 16 inches throughout the building and the openings are then cut out and framed with extra pieces, while in another method the openings are framed first with a double stud at each side and the single studs are then framed in 16 inches apart between these openings. See Art. **179.**

(53) (*a*) No.

(*b*) Because it is very brittle, splits easily, is very likely to be shaky, and is not durable. See Art. **45.**

(54) A fish joint is one where the two pieces of scantling to be joined are butted together, end to end, and secured in place by means of a board spiked to each side of the timbers over the joint. See Art. **87.**

(55) Parasitic plants which derive their nutriment from some other plant or from cut timber, and thereby reduce its vitality. See Art. **27.**

(56) Because they give ventilation to the under side of the shingles and prevent the accumulation of moisture. See Art. **157.**

(57) It should be piled in high and dry locations, kept well up from the ground on staging, with narrow strips placed between the layers to permit a free circulation of air. See Arts. **28** and **37.**

(58) So that when it sags in the center from any cause, it will not disturb the brickwork in the wall above it, even if it falls out of the wall entirely. See Art. **226.**

(59) The butt joint. See Art. **75.**

(60) A mortise joint is one where the two pieces of timber are joined together and maintained in position by means of a tenon or tongue, projecting from one piece into a mortise or socket, which is cut out of the other. The mortise is usually cut out of the center of the timber, and is equal in width to $\frac{1}{3}$ the width of the timber. The tenon is cut to fit the mortise exactly and is also in the center of the timber. When the two pieces are of different thicknesses, the joint is usually framed so as to bring one face of each timber flush with the other. See Art. **76.**

(61) One consists of a large number of small timbers, such as studs, braces, beams, bridging, etc., and the other—the slow-burning system—consists of a small number of large timbers, such as columns and beams, without studs, braces, or bridging between. See Arts. **217** and **218.**

(62) They are notched on their under sides to fit over the sill or the girth, and any irregularity in depth is removed when the notch is cut so that all the beams are the same depth from the top to the bearing surface. See Art. **108.**

(63) To make the thrust against each side of the rafter equal and thereby prevent it from bulging out of line. See Art. **181.**

(64) (*a*) They represent the sizes of the finished openings.

(*b*) To these dimensions there must be added 5 inches in height and 7 inches in width for windows with weight boxes, or, if without weights, an allowance of 3 inches in width will be sufficient. In doors an allowance of 5 inches in width and 3 inches in height should be made. See Art. **130.**

(65) (*a*) Flashing is a term given to all sheet-metal work used in connection with roof covering to insure a water-tight condition at joints and angles.

(*b*) It is required on hips and valleys, ridges and eaves, around scuttles, skylights, gables, flanks of dormer-windows, etc. See Art. **161.**

(66) (*a*) Pieces of old brieks used for filling in partitions.

(*b*) When it is desired to elose the passageway between the studs of the outside walls, to prevent drafts and the consequent rapid spread of flames. See Art. **184.**

(67) They are nailing strips equal to the plaster in thickness plaeed against the studs at various heights to receive the trim or other interior finish and also form a stop for the plaster. See Arts. **123** and **185.**

(68) The system of slow-burning construetion provides a structure where the individual members, sueh as beams, girders, etc., are few in number, but so large in size that they would retain suffieient strength to carry the load imposed upon them even after they have been burned $\frac{1}{3}$ through. See Arts. **218** and **219.**

(69) White eedar. See Art. **46.**

(70) To insulate them from the heat which they would otherwise absorb and likely beeome so hot as to bend and throw the walls or columns out of plumb. See Art. **229.**

(71) (*a*) They are rifts or eracks whieh radiate from the eenter of the log.

(*b*) Caused by shrinkage of the timber incidental to loss of vitality. See Art. **31.**

(72) One square inch of leader section to 75 square feet of roof area. See Art. **160.**

(73) (*a*) Beveled siding consists of sawed and planed boards in commercial lengths of about 16 feet and widths of from 4 inches to 6 inehes, while the thickness is $\frac{5}{8}$ ineh at the lower edge and bevels back to $\frac{3}{8}$ inch at the upper edge. Novelty siding has a uniform thickness of about $\frac{7}{8}$ inch, except where it is rabbeted at the bottom to receive the chamfered edge of the next board below it, and where it is chamfered to fit the rabbet of the next board above it.

(*b*) Beveled siding is the better. See Art. **187.**

(74) Furring strips are small pieces of timber, 1 in.×2 in. in section, which are nailed to the under side of floorbeams to bring them to an even alinement. Furring strips are also used to isolate the plastering from the surface of a wall, thus forming an air space. See Art. **126.**

(75) To make a building strong enough to stand any ordinary stress even after its timbers are partly burned; to make the floor sufficiently water-tight that when a fire gets started in one story the water poured in to quench it will not run through and ruin goods on the floor below; to avoid any corners or pockets where fire could get started without being immediately discovered; and, above all, to provide a building where every part is easily accessible and the fire could be attacked at close quarters, and extinguished, without flooding the entire structure. See Art. **219.**

(76) Cypress and white cedar. See Arts. **46** and **48.**

(77) (*a*) The short beam framed into and supported by two trimmer beams.

(*b*) Two heavy beams spaced one each side of a floor opening to receive the header on which the tail-beams rest.

(*c*) Those beams in a floor system, one or both of whose ends rest upon a header which transmits their loads to other beams, called trimmers, which are made especially heavy to support them.

(*d*) With mortise-and-tenon joints or occasionally with stirrup irons. See Arts. **79, 80,** and **110.**

(78) The disintegration of the fibers, due to the attack of parasitic plants. See Art. **27.**

(79) Over stiffened wire lath laid directly against the woodwork of the partition. See Art. **232.**

(80) (*a*) They are small roof beams, used when the main roof is supported by trusses, or heavy and widely spaced rafters, in order to provide members to which the roofing may be secured.

(*b*) Draw a plan of the purlin in its actual position on the roof and intersect the end of it with a diagonal line at the angle of the hip or valley rafter. From the first point of intersection of this valley-rafter line, draw a line across the plan of the purlin at right angles to its sides, and measure the distance from this last line to the points where the valley rafter intersects the corners of the purlin. Now, if these last distances are laid off successively on three adjacent corners of the purlin itself, and the latter is sawed to a line connecting them, the cut will be the miter required. See Art. **150.**

JOINERY.

(QUESTIONS 1–60. SEC. 10.)

(1) See Art. **93.**

(2) See Art. **96.**

(3) See Art. **86.**

(4) See Art. **76.**

(5) See Art. **65.**

(6) See Art. **58.**

(7) See Art. **47.**

(8) See Arts. **31** and **32.**

(9) See Art. **25.**

(10) See Art. **7.**

(11) See Art. **95.**

(12) See Art. **93.**

(13) (*a*) See Art. **85.**

(*b*) The method of bending and gluing together several layers. See Art. **89.**

(14) See Art. **75.**

(15) See Art. **62.**

(16) Those hinged at the sides, hung in pairs, and closing in the center. See Art. **63.**

(17) See Art. **44.**

(18) The preparation of a measuring rod. See Art. **31.**

(19) (*a*) See Art. **23.**
(*b*) See Art. **24.**

(20) See Art. **11.**

(21) See Arts. **110** and **111.**

(22) See Art. **92.**

(23) See Art. **71.**

(24) See Art. **62.**

(25) . 50 square feet. See Art. **53.**

(26) See Art. **43.**

(27) See Art. **30.**

(28) See Art. **20.**

(29) See Art. **11.**

(30) See Art. **105.**

(31) See Art. **91.**

(32) The method to be employed is similar to that described in Art. **71.**

(33) See Art. **62.**

(34) See Art. **53.**

(35) See Art. **42.**

(36) See Art. **28.**

(37) See Art. **1.**

(38) For the fronts of drawers. See Art. **14.**

(39) See Art. **98.**

(40) See Art. **91.**

(41) The method is clearly outlined in Art. **81.**

(42) See Art. **70.**

(43) See Art. **60.**

(44) See Art. **52.**

(45) See Art. **33.**

(46) See Art. **26.**

(47) See Art. **2.**

(48) See Art. **14.**

(49) See Art. **99.**

(50) By the rule given in Art. **90,** the thickness is $.05 \times$ radius of curvature of bend. In this case the radius is $15 \times \frac{1}{2} = 7\frac{1}{2}$ feet. Hence, thickness $= 7\frac{1}{2} \times .05 = .375$ in. $= \frac{3}{8}$ in. Ans.

(51) The method is shown in Art. **78.**

(52) See Arts. **66** and **67.**

(53) See Art. **57.**

(54) See Art. **51.**

(55) See Art. **36.**

(56) See Art. **26.**

(57) See Art. **5.**

(58) See Art. **84.**

(59) See Art. **88.**

(60) See Art. **114.**

STAIR BUILDING.

(QUESTIONS 1-42. SEC. 11.)

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(1) (*a*) See Art. **26.**
(*b*) See Art. **28.**

(2) (*a*), (*b*), and (*c*) See Art. **3.**

(3) (*a*), (*b*), and (*c*) See Art. **19.**

(4) (*a*) and (*b*) See Art. **3.**

(5) (*a*) and (*b*) See Art. **15.**

(6) See Art. **42,** Fig. 34.

(7) See Art. **13.**

(8) (*a*) See Art. **48.**
(*b*) See Art. **51.**

(9) (*a*) and (*b*) See Art. **9.**

(10) (*a*) and (*b*) See Art. **17.**

(11) (*a*) See Art. **11.**
(*b*) See Art. **12.**

(12) See Art. **3.**

(13) (*a*) See Art. **7.**

(*b*) By rule I, the width of tread is $66 \div 7 = 9\frac{3}{7}$ inches; by rule II, the width of tread is: $7 + 5 = 12$, and $5 \times 2 = 10$ inches.

(14) See Art. **43,** Fig. 37.

(15) (*a*) From 6 to 7 inches.
(*b*) See Art. **6.**

(16) See Art. **60.**

(17) See Art. **16.**

(18) Distance $b\,c = 24 - 9 = 15$ in.; height $c\,e = \frac{15}{24}$ of 11 in. $= 6\frac{7}{8}$ in., which should be the distance scaled on diagram.

(19) See Art. **44.**

(20) (*a*) and (*b*) See Art. **61.**

(21) (*a*) and (*b*) See Art. **5.**

(22) (*a*) and (*b*) See Art. **14.**

(23) (*a*) and (*b*) See Art. **19.**

(24) See Art. **41,** Fig. 33.

(25) (*a*) and (*b*) See Art. **5.**

(26) See Art. **10.**

(27) See Art. **25.**

(28) (*a*) and (*b*) See Art. **9.**

(29) (*a*) and (*b*) See Art. **18.**

(30) See Art. **2.**

(31) (*a*) and (*b*) See Art. **8.**

(32) (*a*) and (*b*) See Art. **4.**

(33) See Art. **24.**

(34) See Art. **8.**

(35) See Art. **32,** Fig. 23 (*d*)

(36) (*a*) and (*b*) See Art. **4.**

(37) See Art. **1.**

(38) (*a*) and (*b*) See Art. **1.**

(39) (*a*) and (*b*) See Art. **20.**
(*c*) See Art. **22.**

(40) See Art. **39,** Fig. 30.

(41) (*a*) See Art. **53.**
(*b*) See Art. **56.**

(42) (*a*) and (*b*) See Art. **5.**

8-13

ORNAMENTAL IRONWORK.

(QUESTIONS 1–41. SEC. 12.)

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(1) See Art. **75.**

(2) See Art. **44.**

(3) See Art. **1.**

(4) See Art. **26.**

(5) See Art. **58.**

(6) See Arts. **6** and **7.**

(7) See Art. **18.**

(8) See Art. **61.**

(9) See Art. **3.**

(10) See Art. **37.**

(11) See Art. **72.**

(12) See Art. **6.**

(13) See Arts. **30** and **31.**

(14) See Art. **10.**

(15) See Art. **1.**

(16) See Art. **28.**

(17) See Art. **1.**

(18) See Art. **4.**

(19) See Art. **37.**
(20) See Art. **4.**
(21) See Art. **2.**
(22) See Art. **22.**
(23) See Art. **61.**
(24) See Art. **9.**
(25) See Art. **29.**
(26) See Arts. **69** and **70.**
(27) See Art. **4.**
(28) See Art. **34.**
(29) See Art. **75.**
(30) See Art. **4.**
(31) See Art. **23.**
(32) See Art. **61.**
(33) See Art. **8.**
(34) See Art. **29.**
(35) See Art. **62.**
(36) See Art. **5.**
(37) See Art. **58.**
(38) See Art. **61.**
(39) See Art. **4.**
(40) See Art. **23.**
(41) See Art. **35.**

ROOFING.

(QUESTIONS 1–52. SEC. 13.)

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(1) See Arts. **144** and **145.**

(2) See Art. **25.**

(3) See Art. **147.**

(4) See Art. **42.**

(5) See Art. **136.**

(6) Solder is an alloy of two or more metals, which melts at a comparatively low temperature. It is used to make a metallic connection between two metal sheets or plates.

(7) The gutter flashing should be of 16-ounce copper. See Art. **132.**

(8) See Fig. 51, Art. **61.**

(9) Because of their cheapness and lightness. See Art. **80.**

(10) *Rolls* when laid with the pitch, and *drips* when laid at right angles to the pitch. See Art. **100.**

(11) The length of the nail should be slightly less than the combined thicknesses of the roofing material and roof boards, so that it will not protrude below the lower surface of the sheathing.

(12) The thickness varies from $\frac{1}{8}$ to $\frac{1}{4}$ inch, the common average being $\frac{3}{16}$ inch. See Art. **114.**

(13) (*a*) See Art. **14.**
(*b*) See Art. **14.**

(14) The first course. See Art. **120.**

(15) See Art. **18.**

(16) See Art. **61.**

(17) See Art. **117.**

(18) See Arts. **88** to **91.**

(19) See Art. **104.**

(20) See Art. **9.**

(21) See Art. **90.**

(22) (*a*) Six.
(*b*) In exposed locations. See Art. **57.**

(23) See Art. **133.**

(24) See Art. **31.**

(25) See Art. **48.**

(26) See Arts. **22** to **24.**

(27) See Art. **72.**

(28) See Art. **11.**

(29) See Fig. 12 (*c*), Art. **9.**

(30) See Art. **51.**

(31) See Art. **4.**

(32) See Art. **45.**

(33) See Art. **74.**

(34) Iron coated with zinc. See Art. **85.**

(35) Clips should not be more than 20 inches apart. See Art. **100.**

(36) See Art. **115.**

(37) The solder will flow more readily than on copper. See Art. **75.**

(38) One inch per foot. See Art. **99.**

(39) See Art. **37.**

(40) See Art. **128.**

(41) See Art. **31.**

(42) See Art. **63.**

(43) For the sake of economy in time and labor. See Art. **78.**

(44) See Art. **86.**

(45) See Art. **98.**

(46) See Art. **107.**

(47) See Art. **113.**

(48) Because of the danger of galvanic action. See Art **107.**

(49) See Art. **119.**

(50) See Art. **56.**

(51) Muriatic acid. See Art. **109.**

(52) ¼ inch per foot. See Art. **103.**

SHEET-METAL WORK.

(QUESTIONS 1-60. SEC. 14.)

(1) See Art. **10** and Fig. 6.

(2) See Art. **16** and Fig. 8.

(3) See Art. **20.**

(4) See Art. **28.**

(5) See Art. **35.**

(6) (*a*) See Art. **42** and Fig. 29.

(*b*) See Art. **42** and Fig. 30.

(7) See Art. **58.** Allow 1 square inch sectional area of leader for each 75 square feet roof area; hence, the number of pipes is

$$\frac{100 \times 100}{75 \times 4^2 \times .7854} = 11 \text{ pipes.} \quad \text{Ans.}$$

(8) See Art. **65.**

(9) See Art. **75.**

(10) See Arts. **3** and **4.**

(11) See Art. **12.**

(12) See end of Art. **16.**

(13) See Art. **21.**

(14) See Art. **30.** The metal work should be nailed across the grain of the wood, not with it.

(15) See Arts. **36** and **37.**

(16) See Arts. **43** and **44.**

(17) See Art. **59.**

(18) See Art. **68.**

(19) (*a*) See Arts. **77** and **78.**
(*b*) See "Cold-Rolled Copper," Art. **79.**
(*c*) See Art. **78.**

(20) See Art. **5.**

(21) See Art. **12.**

(22) See Art. **18.**

(23) See Art. **22** and Fig. 14.

(24) See Art. **31.**

(25) See Art. **38.**

(26) See Arts. **45** and **46.**

(27) See Art. **61.**

(28) See Art. **69** and Fig. 55.

(29) (*a*) See Arts. **79** and **80.**
(*b*) and (*c*) See Art. **81.**

(30) See Art. **6.**

(31) See Arts. **14** and **15.**

(32) See Art. **18.**

(33) See Art. **24.**

(34) See Art. **32.**

(35) See Art. **39.**

(36) See Arts. **48** and **49,** also Figs. 37 and 39.

(37) See Art. **62.**

(38) See Arts. **71** and **72.**

(39) See Art. **82.**

(40) See Art. **7.**

(41) See end of Art. **14.**

(42) See Art. **19.**

(43) See Art. **26.**

(44) See Art. **33.**

(45) See Art. **40.**

(46) See Arts. **51** and **52.**

(47) See Art. **63.**

(48) See Art. **73.**

(49) With an elastic oil cement. They should be pointed only when stones are thoroughly dry and warm. See Arts. **48** and **49.**

(50) See Art. **8** and Fig. 4.

(51) See Art. **16.**

(52) See "Cup Joints," Art. **20.**

(53) See Art. **27.**

(54) See Art. **33.**

(55) See Art. **41.**

(56) (*a*) See Arts. **54** and **55.**
(*b*) See Art. **56.**

(57) See Art. **64.**

(58) See Art. **74.**

(59) See Art. **79.**

(60) See Art. **9** and Fig. 5.

Electric-Light Wiring and Bellwork.

(QUESTIONS 1–56. SEC. 15.)

(1) See Art. **4.**

(2) See Art. **14.**

(3) The multiple-arc, or parallel, system, the multiple-series system, and the three-wire system.

(4) The ampere, the volt, and the ohm.

(5) See Art. **75.**

(6) A steady current is maintained through the magnet coils of a vibrating bell, which, when interrupted, allows the bell to ring from an open-circuit battery.

(7) The number of lamps N is 30; the distance F is 100 feet; the drop in potential E is 1.25 volts. By formula **5,** the resistance per foot of the wire

$R_f = \frac{1.25}{30 \times 100} = .000417$ ohm per foot, or .417 ohm per 1,000 feet.

This corresponds to No. 6 B. & S. gauge. The current taken is $.5 \times 30 = 15$ amperes, which is safe; see Table 1. Ans.

(8) See Art. **3.**

(9) The resistance per 1,000 feet is the given resistance divided by $\frac{3540}{1000} = 3.54$.

$$\frac{2.917}{3.54} = .824 \text{ ohm.}$$

By reference to Table 1, the size of wire corresponding to this resistance is No. 9. Ans.

(10) See Art. **24.**

(11) See Art. **1.**

(12) $6\frac{3}{8}$ in. $= 6.375$ in. $= 6{,}375$ mils. $6{,}375^2 = 40{,}640{,}625$ circular mils. Ans.

(13)

3 lamps of 50 c. p.	=	9 of 16 c. p.
8 lamps of 32 c. p.	=	16 of 16 c. p.
32 lamps of 16 c. p.	=	32 of 16 c. p.
Total	=	57 lamps = N.

The drop in potential E is 5 volts, and the distance F is 340 feet. Then, by formula **5,** Art. **61,** the resistance per foot of the feeder will be

$$R_f = \frac{5}{57 \times 340} = \frac{5}{19{,}380} = .000258 \text{ ohm per foot} = .258 \text{ ohm per 1,000 feet.}$$

This corresponds to No. 4 B. & S. wire; see Table 1. Ans.

(14) See Art. **74.**

(15) When it is required to ring a bell continuously after the push button is pressed.

(16) See Art. **103.**

(17) (*a*) The initial voltage is first determined. By formula **10,** Art. **70,** $V' = \frac{100 \times 110}{95} = 115.79$ volts. By formula **11,** the drop in potential $E = 115.79 - 110 = 5.79$ volts. The current C is 13.5 amperes. By Ohm's law, resistance of the two conductors $= R = \frac{E}{C} = \frac{5.79}{13.5} = .43$ ohm. Ans.

(*b*) The resistance per foot $= \frac{.43}{2 \times 320} = .00067$ ohm. Ans.

(18) They may be used for a few lamps, such as those on an electrolier, or the lights in a small room.

(19) See Art. **47.**

(20) The number of lamps N is 65; the distance F is 540 feet; the drop E is 4.5 volts. Since the voltage of the lamps is 110, and they are of 16 c. p., the resistance per foot of the conductor is, by formula **5,** Art. **61,**

$R_f = \frac{4.5}{65 \times 540} = .000128$ ohm per foot $= .128$ ohm per 1,000 feet.

From the wire table, we find this to correspond with No. 1 B. & S. Ans.

(21) The weight of copper required is only $\frac{3}{8}$ of that necessary for the two-wire system.

(22) See Fig. 56, Art. **77.**

(23) A battery of about six cells in series with a grounded spark coil.

(24) The resistance per 1,000 feet of No. 6 wire is, by Table 1, .411 ohm; or, per foot, .000411 ohm $= R_f$. The four 32 c. p. lamps = eight 16 c. p. lamps, and the total number N is $45 + 8 = 53$. The distance F from the dynamo $= 165$ feet. Then, by formula **8,** the drop is

$$E = .000411 \times 53 \times 165 = 3.59 \text{ volts.} \quad \text{Ans.}$$

(25) See Art. **44.**

(26) See Arts. **49, 54, 56.**

(27) The current required for the lamps is 36 amperes, since each 32 c. p. lamp takes 1 ampere. By reference to Table 1, it will be seen that No. 8 wire is not allowed to carry a larger current than 25 amperes, and it will therefore be too small, and must not be used. Ans.

(28) The location of each box should be so chosen that the different circuits may all have a uniform fall of potential.

(29) See Art. **78.**

(30) (*a*) No. 18 B. & S. gauge. (*b*) No. 14 or 16 B. & S.

(31) The circuit through any one lamp has a length of 166 feet, and the single distance F is $\frac{166}{2} = 83$ feet; the number of lamps N is 32. In finding the actual drop in volts, we make use of formula **10:**

$$V' = \frac{100 \times 110}{98.5} = 111.7 \text{ volts.}$$

Then, by formula **11,** the drop is $E = 111.7 - 110 = 1.7$ volts. The resistance per foot of the conductor will be, by formula **5,**

$$R_f = \frac{1.7}{32 \times 83} = .000641 \text{ ohm per foot} = .641 \text{ ohm per 1,000 feet.}$$

The nearest size to this is No. 8, and the current being 16 amperes, this wire will be safe. Ans.

(32) See Art. **12.**

(33) Glass or porcelain insulating tubes must be secured in the wall, and the wires should be passed through them.

(34) The initial voltage, by formula **10,** Art. **70,** is

$$V' = \frac{190 \times 55}{100 - 2.5} = 56.45 \text{ volts.}$$

The drop in potential is, by formula **11,**

$$E = 56.45 - 55 = 1.45 \text{ volts.}$$

The number of lamps is 10 of 16 c. p., and 3 of 32 c. p. = 16 lamps of 16 c. p. The distance F is 110 feet. Then, by formula **6,** the resistance per foot is

$$R_f = \frac{1.45}{2 \times 16 \times 110} = .000414 \text{ ohm per foot} = .414 \text{ ohm per 1,000 feet.}$$

This corresponds to No. 6 wire, which will safely carry the current, 32 amperes. Ans.

(35) See Art. **79.**

(36) The circuit may include a relay, which will ring an alarm bell when the system is short-circuited.

(37) The number of lamps $N = 240$; the distance F is 135 feet; the drop E is 2 volts. By formula **9**, the area of the wire is

$$A = \frac{10.8 \times 240 \times 135}{2} = 174{,}960 \text{ circular mils.}$$

This size lies between No. 000 and No. 0000, and will, therefore, carry over 145 amperes; but the current is only 120 amperes, and the cable will be of safe capacity. Ans.

(38) See Art. **19.**

(39) See Art. **39.**

(40) The resistance per 1,000 feet of No. 6 wire is .411 ohm, or, per foot, .000411 ohm $= R_f$; the number of lamps N is 25; the distance F is 225 feet. Then, by formula **7**, the drop on the line is

$$E = 2 \times .000411 \times 25 \times 225 = 4.625 \text{ volts.} \quad \text{Ans.}$$

(41) See Art. **82.**

(42) The insulation of the wires must be very perfect, and they should be well protected whenever they approach any grounded metal work.

(43) See Art. **21.**

(44) The resistance per 1,000 feet of No. 4 wire is .259 ohm, or, per foot, .000259 ohm $= R_f$; the number of lamps N is 46; the distance F is 84 feet. Then, by formula **8**, the drop in potential is

$$E = .000259 \times 46 \times 84 = 1.00 \text{ volt.} \quad \text{Ans.}$$

(45) See Art. **83.**

(46) See Art. **42.**

(47) The 40 lamps present an equivalent of $3 \times 40 = 120$ lamps of 16 c. p.; the length of feeder is 85 feet; the drop

in potential is 1.25 volts. Then, by formula **5**, the resistance per foot of the feeder is

$$R_f = \frac{1.25}{120 \times 85} = .000123 \text{ ohm per foot} = .123 \text{ ohm per } 1{,}000 \text{ feet.}$$

This corresponds near enough to No. 1 wire. The current being 1.5 amperes per lamp $= 1.5 \times 40 = 60$ amperes, which is within the allowable limit, and is safe. Ans.

For the loop, the distance to be considered is one-half that measured round the dome. See Art. **67.** The diameter of the dome is 120 feet, and the circumference is, therefore, 377 feet. The lamp feet will then be $120 \times \frac{377}{2} = 22{,}620$, and the drop of potential is 2.5 volts. Then, by formula **5**, the resistance of the conductor per foot is

$$R_f = \frac{2.5}{22{,}620} = .00011 \text{ ohm per foot} = .11 \text{ ohm per } 1{,}000 \text{ feet.}$$

The nearest size of wire is No. 0 B. & S. Ans.

(48) See Figs. 26, 27, and 29.

(49) The number of lamps $N = 66$; the distance F is 180 feet; the drop E is 2.5 volts. By formula **12**, the resistance per foot of the wire is

$$R_f = \frac{2 \times 2.5}{66 \times 180} = .00042 \text{ ohm per foot} = .42 \text{ ohm per } 1{,}000 \text{ feet.}$$

This corresponds to No. 6 wire, and since the current will be $\frac{66}{4} = 16.5$ amperes, this size will be safe. Ans.

(50) All but one must be changed to give a single stroke.

(51) See Art. **40.**

(52) By formula **10**, the initial voltage is

$$V' = \frac{100 \times 110}{100 - 1.25} = 111.4 \text{ volts.}$$

By formula **11**, the drop in potential is

$$E = 111.4 - 110 = 1.4 \text{ volts.}$$

Since it is evident that a large conductor is required, its area is calculated by formula **9**;

$$A = \frac{10.8 \times 480 \times 80}{1.4} = 296{,}000 \text{ circular mils.}$$

By interpolation in Table 2, it will be found that this cable will carry 269 amperes, and it is therefore safe. Ans.

(53) See Art. **87**, Fig. 66.

(54) By formula **10**, the initial voltage is

$$V' = \frac{100 \times 55}{100 - 4.6} = 57.65 \text{ volts.}$$

By formula **11**, the drop in voltage is

$$E = 57.65 - 55 = 2.65 \text{ volts.} \quad \text{Ans.}$$

(55) See Art. **52**.

(56) According to formula **10**, the initial voltage is

$$V' = \frac{100 \times 110}{97} = 113.4 \text{ volts.}$$

By formula **11**, the drop is

$$E = 113.4 - 110 = 3.4 \text{ volts} = E.$$

The number of lamps N is 84; the distance F is 320 feet. Then, by formula **12**, the resistance of the wire per foot is

$$R_f = \frac{2 \times 3.4}{84 \times 320} = .000253 \text{ ohm per foot} = .253 \text{ ohm per } 1{,}000 \text{ feet.}$$

The nearest size of wire is No. 4, which will carry 50 amperes. Since the current taken by the lamps is only one-half of that required in the ordinary two-wire system, it will be $\frac{84 \times .5}{2} = 21$ amperes, and the wire will be safe. Ans.

PLUMBING AND GAS-FITTING.

(QUESTIONS 1-100. SEC. 16.)

(1) See Art. **35.**

(2) See Art. **56** and Fig. 27.

(3) See Arts. **81** and **82.**

(4) See Art. **121.**

(5) See Art. **151.**

(6) See Art. **178** and Fig. 64.

(7) See Table 9, Art. **209.** The increase of pressure for each 10 feet in rise is .073 inch of water. Consequently, the increase in pressure at the 18th floor will be $.073 \times \frac{220}{10} = 1.6$ inches of water. The total pressure at the 18th floor will be $1.6 + 1.25 = 2.85$ inches of water. Ans.

(8) See Arts. **239** and **240.**

(9) See Table 11, Art. **260.** In this table, the distance, 20 inches, shows a candlepower of 16. Or, since the intensity of light is inversely as the square of the distance from the source of light, it follows that the candlepower of the gas flame will be $80^2 \div 20^2 = 16$ times greater than that of the candle flame. The candle flame, being 1 candlepower, however, it also follows that the gas flame is 16 candlepower.

Ans.

(10) See Arts. **4** to **9.**

(11) See Art. **36.** Long hoppers are used for outside closets only, because the traps must be buried below frost line.

(12) See Art. **57** and Fig. 28.

(13) See Art. **90.**

(14) See Art. **123.**

(15) (*a*) See Art. **154.**

(*b*) See Art. **155.** Put on a swing check; also vent the sewer; also, if possible, provide periodical flushing apparatus.

(16) (*a*) See Art. **179.**
(*b*) See Arts. **180** and **181.**
(*c*) See Art. **182.**

(17) See Table 10. $2\frac{1}{2}$-inch pipe. Ans.

(18) See Arts. **241** and **242.**

(19) (*a*) See "Shields," Art. **248.**
(*b*) Yes.

(20) See Arts. **10** to **12.**

(21) See Fig. 11, Art. **14.** Since a running trap is to be used, it would be connected up as shown in Fig. 1. It is vented at *a*. Guard against placing the trap between the overflow pipe *b* and the waste outlet to the bath. The pipes passing through the safe *c* should have flange joints wiped as at *d*, *e*, and *f* to make the safe water-tight. A safe waste pipe *g*

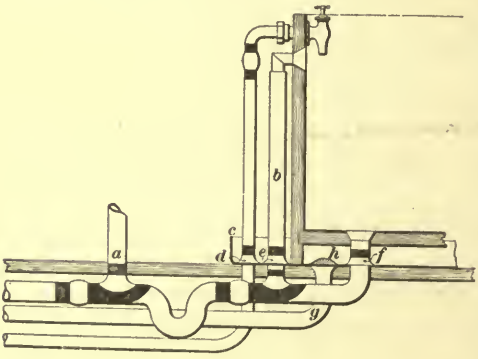


FIG. 1.

furnished with a raised brass strainer *h* is provided to carry off any leakage to a safe point.

(22) (*a*) See Art. **58.** Usually where there is no space available to stand a vertical boiler.

(*b*) The return pipe to the waterback should connect to bottom of boiler in order to feed the waterback with the coldest water; also, to cause all the water in boiler to be heated and prevent a stratum of cold water from lying "dead" in the bottom of the boiler. The feed to the boiler should also join at or near the bottom; otherwise, the cold water entering the boiler will cool the hot water too much by mixing with it. An improved method of connecting is shown in Fig. 2. The arrows show the direction of the main currents when hot water is being drawn at the fixtures.

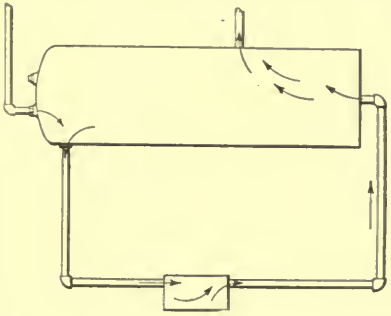


FIG. 2.

(23) (*a*) See Arts. **102** and **105.**
(*b*) See Arts. **103** and **106.**

(24) See Art. **124.**

(25) See Art. **160.**

(26) See Arts. **185** and **186.**

(27) (*a*) See Art. **214.**
(*b*) and (*c*) See Art. **215,** also Art. **216** and Fig. 77.

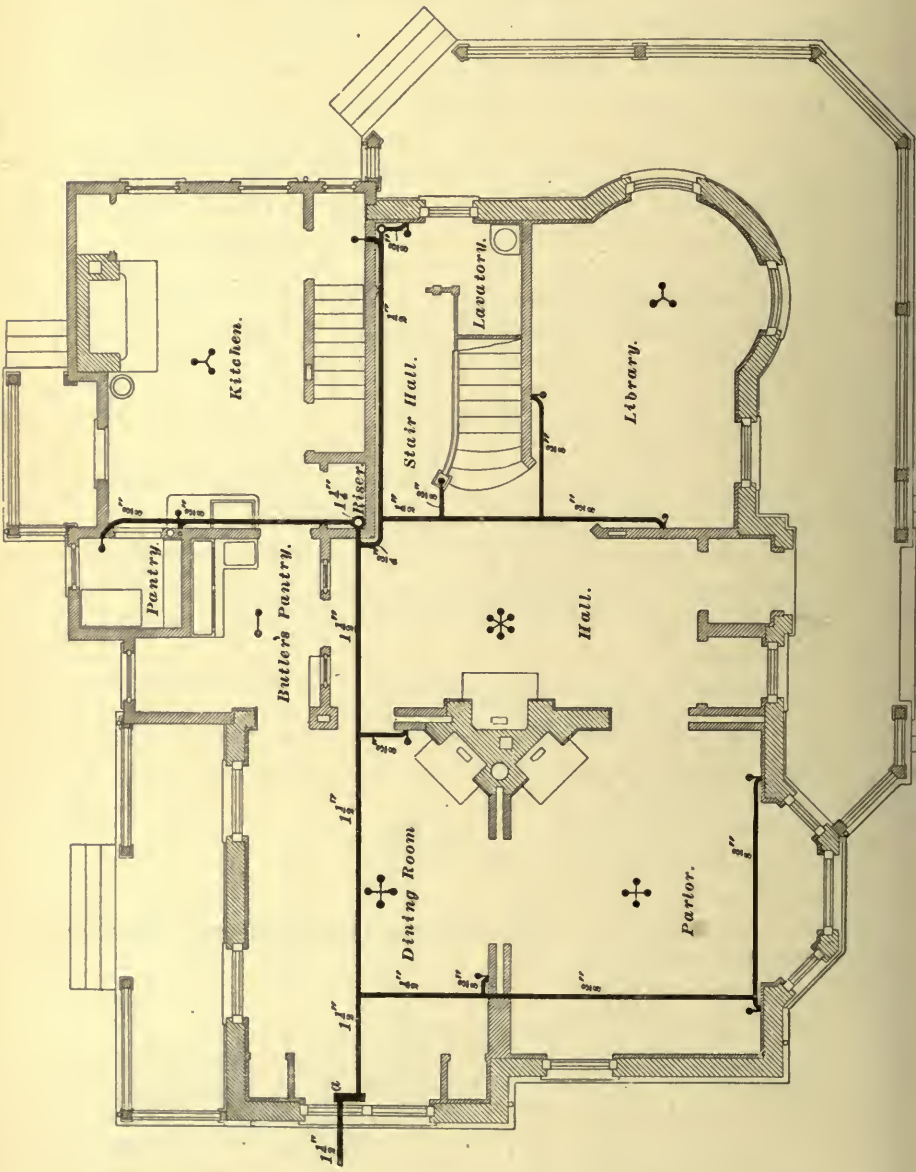
(28) See Art. **244.**

(29) See Art. **251.**

(30) See Art. **13.**

(31) See Art. **38** and Fig. 20. Use a brass floor flange.

(32) See Art. **57.** Run a circulation pipe from the highest point of the hot-water supply pipe to basin. This point


Kitchen.
Pantry.
Butler's Pantry.
Stair Hall.
Lavatory.
Riser.
Library.
Hall.
Dining Room
Parlor.

must be near the basin in order to obtain hot water the instant the basin cock is opened.

(33) See Art. **108,** also the trap at *A* in Fig. 34

(34) See Art. **125.**

(35) See Art. **161.**

(36) See Art. **148.**

(*a*) A little more calking will remedy the defect.

(*b*) The pipe or fitting should be renewed.

(*c*) The pipe should be renewed.

(*d*) The flaw should be removed; that is, the casting should be renewed.

(37) Fig. 3 shows how the piping in the basement may be arranged, also the sizes of the several pipes. Since the riser to the second floor is not changed, the piping shown in Fig. 81, *Plumbing and Gas-Fitting*, will remain unchanged. Note the newel-post branch.

(38) See Art. **245.**

(39) See Arts. **203** and **204.** (*a*) Acetylene.

(*b*) It is made from calcium carbide and water.

(*c*) No. It will smoke. The burners must have special thin slits to spread the flame.

(40) See Art. **17.**

(41) See Arts. **40** and **41.**

(42) See Art. **59.** (*a*) Should be made of copper, tinned on the surfaces in contact with the water. A double boiler is used, because one waterback only will do the heating.

(*b*) This sketch will be similar to Fig. 30, *Plumbing and Gas-Fitting*, Art. **59.** Guard against placing sediment cocks *L* and *M* in such a position that the inner boiler can be emptied while the outer one is full.

(43) See Arts. **109** and **110.**

(44) See Art. **128.**

(45) See Art. **167.** In order to find how many cubic feet of water have passed through the meter since it was last read, we subtract the last reading from this; thus, $47,805 - 6,417 = 41,388$. Ans.

(46) See Art. **163.** Since the man who will operate the pump is a gardener, and, consequently, not familiar with machinery, a simple and safe engine should be employed. A double-cylinder hot-air engine is especially adapted for this work, and is extensively employed when the height the water has to be raised exceeds 75 or 100 feet.

(47) See Art. **221.**

(48) See Arts. **246** and **247.**

(49) See Art. **257.**

(50) See Arts. **15, 16,** and **18.**

(51) See Art. **45.** Set the supply cock so that the pan will tilt every 5 minutes.

(52) See Arts. **62** and **63.**

(53) See Art. **114.**

(54) See Art. **129.**

(55) Since the bath branch and the sink branch are each

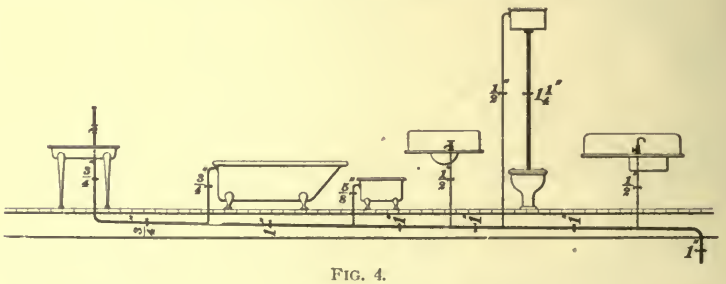


FIG. 4.

$\frac{3}{4}$ inch in diameter, the 1-inch pipe should be run along as shown, and reduced only for the sink. See Fig. 4.

(56) See Arts. **197, 199,** and **200.**

(57) (*a*) No. See Art. **224.**

(*b*) With non-conducting materials to keep in the heat.

(*c*) Guard against contact between the pipe and the wire. Insulate the pipe with rubber tape, as explained.

(58) See Art. **249.**

(59) See Art. **259.**

(60) See Art. **19.**

(61) See Arts. **48** and **49,** also Fig. 25, *Plumbing and Gas-Fitting.*

(62) See Art. **68.** (*a*) About $30 \times 60 \times 2 = 3{,}600$ gal. Ans.

(*b*) About $30 \times 60 \times 5 = 9{,}000$ gal. Ans.

(63) See Art. **115.** Use a back-water trap, as shown in Fig. 39, *Plumbing and Gas-Fitting*, or place a check-valve on the branch.

(64) (*a*) 275 feet per minute.

(*b*) 1 in 40. See Arts. **130** and **131.**

(65) See Art. **170** and Fig. 63.

(66) (*a*) See Art. **203.**

(*b*) See Arts. **203** and **204.**

(67) See Art. **225.**

(68) See Art. **250.**

(69) See Art. **89.**

(70) See Arts. **22** to **25.**

(71) See Art. **52.**

(72) See Art. **72.**

(73) See Art. **117.**

(74) See Arts. **139** and **141.** It should be located inside the cellar wall and above the floor. Failing that, it should be built in a brick manhole.

(75) There being a tank in the attic, the pressure must be low; hence, see "low pressure" column in Table 7, Art. **172.** Use the larger sizes for the upper floor.

(76) (*a*) See Art. **204.** About 240 candlepower.

(*b*) Since acetylene has about 240 candlepower and common gas about 16 candlepower, the number of gas burners required will be about $\frac{240}{16} = 15$. Ans.

(77) (*a*) See Art. **227.**

(*b*) See Art. **228.**

(*c*) See Arts. **229** and **230.**

(78) (*a*) See Art. **252.** The aim is to illuminate all objects, within the ordinary field of vision, to about the same degree of brilliancy as that afforded by diffused daylight.

(*b*) See Art. **253.** By a large number of small lights located up near the ceiling of the room to be lighted.

(79) See Art. **91.**

(80) In Fig. 5, three tubs are shown fitted up in the same manner as the two shown in Fig. 16, Art. **31,** *Plumbing*

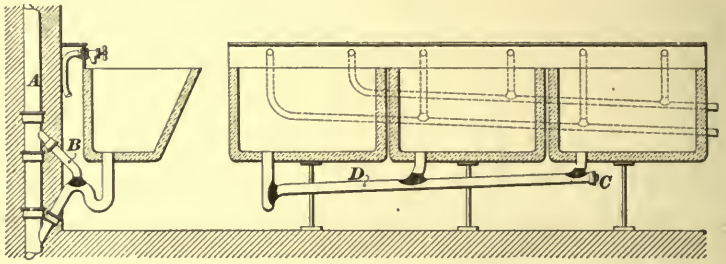


FIG. 5.

and Gas-Fitting, with this difference: The trap will be connected to the outlet of the left-hand tub. The water pipes, as shown by dotted lines, will extend to the third tub. A 3-inch pipe stack will be shown, as at *A*, and a $\frac{3}{4}$ **S** trap will join the stack by means of a **Y** branch and a brass

ferrule connection. The back vent *B* will join the stack in a similar manner, the branch being set above level of bottom of tubs. A trap screw may be wiped in the end *C* to facilitate cleaning the waste pipe *D*.

(81) See Art. **54.**

(82) See Tables 1, 3, and 4, Art. **73.**

$$\begin{aligned} 76\times 3.5\times .06 &= \$\,15.96 \\ 110\times 3.5\times .07 &= 26.95 \\ 96\times 8\times .07 &= 53.76 \\ 56\times \tfrac{9}{16}\times .20 &= 6.30 \\ \text{Total,} &\quad \$\,102.97 \text{ Ans.} \end{aligned}$$

(83) See Arts. **118** and **119.** That shown in Fig. 41 is the most efficient.

(84) See Arts. **142** and **143.**

(85) (*a*) See Art. **173.** By filtration through beds of fine sand, charcoal, etc.

(*b*) By aeration. See Art. **174.**

(86) See Art. **205.**

(87) See Arts. **231** and **232.**

(88) See rule, Art. **254.** Main floor will require $\frac{65\times 90}{40}$ = 146 burners. Each balcony will require $\frac{1200}{40}$ = 30 burners. Total number of burners required = 146 + 30 + 30 = 206. Ans.

(89) See Art. **112.**

(90) See Art. **33.**

(91) See Art. **55.**

(92) See Art. **74.**

(93) See Arts. **120** and **127.** The seal is maintained, and siphonage is prevented by the use of a vent pipe.

(94) (*a*) See Art. **148.**

(*b*) See Art. **149.**

(95) See Art. **176.**

(96) See Art. **206.** Since $4\frac{1}{2}$ gal. of this gasoline will produce 1,000 cu. ft. of 15 c. p. gas, and since ten 5-foot burners will consume $10 \times 100 \times 5 = 5{,}000$ cu. ft. of gas in 100 hours, it follows that $4.5 \times \frac{5000}{1000} = 22.5$ gal. of gasoline are required. Ans.

(97) (*a*) See Art. **233.**
(*b*) See Art. **235.**

(98) See Art. **256.**

(99) (*a*) See Art. **113.**
(*b*) No.

(100) See Art. **34.**

Heating and Ventilation of Buildings.

(QUESTIONS 1–100. SEC. 17.)

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(1) See Table 9, Art. **65.** One ton is 2,000 pounds. The amount of air required for the hot-water boiler will be

$$304.85 \times 2{,}000 = 609{,}700 \text{ cu. ft.} \quad \text{Ans.}$$

The amount of air required for the low-pressure steam boiler (see Table 9) is equal to

$$265.45 \times 2{,}000 = 530{,}900 \text{ cu. ft.} \quad \text{Ans.}$$

The amount required for the power boiler is

$$170.83 \times 2{,}000 = 341{,}660 \text{ cu. ft.} \quad \text{Ans.}$$

(2) See Art. **90.** The heat emitted from each square foot of the pipe surface to air having a velocity of 14 feet per second = 7.5 B. T. U. per hour per degree difference (see Table 14). Consequently, each square foot will emit, on an average,

$$7.5 \times 220 = 1{,}650 \text{ B. T. U. per hour.}$$

The size of the heater then will be

$$\frac{5000000}{1650} = 3{,}030 \text{ sq. ft.}$$

Therefore, $3{,}030 \times 3 = 9{,}090$ feet of 1-inch pipe are required. Ans.

(3) See Art. **117.** No.

(4) See Art. **137.**

(5) See Arts. **168** and **169.**

(6) (*a*) See Art. **195.**
(*b*) See Arts. **196** and **197.**

(7) According to Table 18, a 6-inch pipe will be large enough.

(8) (*a*) See Art. **235.**
(*b*) See Art. **234.**

(9) See Art. **271.**

(10) (*a*) See Arts. **7** to **14,** inclusive.
(*b*) See Arts. **9** and **13.**

(11) See Art. **66.**

(12) See Baldwin's rule, Arts. **92** and **93.** Glass surface is equal to

$$6 \times 3 \times 4 = 72 \text{ sq. ft.}$$

The exposed wall surface, reduced to glass equivalent, is equal to

$$\frac{10\,(35+30)-72}{10} = 57.8 \text{ sq. ft.}$$

Cooling surfaces to be calculated for have an area of

$$57.8 + 72 = 129.8 \text{ sq. ft.}$$

Substituting in the formula, we have,

$$S = \frac{70+10}{227-70} \times 129.8 = 66 \text{ sq. ft., nearly.}$$

To this we add 25%, or 16.5 square feet, for air leakage, which gives

$$66 + 16.5 = 82.5 \text{ sq. ft.}$$

Then we add 20%, or 16.5 square feet, for wind exposure, which gives as the correct size of the radiators

$$82.5 + 16.5 = 99 \text{ sq. ft.}$$

If this is divided into two radiators, they will be as follows:

$$\left.\begin{aligned} a &= 66 \text{ sq. ft.} \\ b &= 33 \text{ sq. ft.} \end{aligned}\right\} \text{Ans}$$

(13) See Art. **116.**

(14) See Art. **138.**

(15) See Arts. **170** and **171.**

(16) See Arts. **198** and **207.**

(17) See Table 21, Art. **223.** Total amount of glass surface and its equivalent equals

$$2{,}400 + \tfrac{600}{4} = 2{,}550 \text{ sq. ft.}$$

According to the table of ratios, the heating surface is

$$\frac{2{,}550}{3.5} = 730 \text{ sq. ft., nearly.} \quad \text{Ans.}$$

(18) See rule **13**, Art. **241.**

Glass surface = 50 sq. ft.
Glass equivalent of wall surface (240 sq. ft.) = 60 sq. ft.
Total, 110 sq. ft.

Area of pipe for first floor is 110×1.5
= 165 sq. in., corresponding to $14\frac{1}{2}$ in. diam.
Area of pipe for second floor is 110×1.25
= 137 sq. in , corresponding to $13\frac{1}{4}$ in. diam.
Area of pipe for third floor is 110×1
= 110 sq. in., corresponding to $11\frac{7}{8}$ in. diam.
} Ans.

(19) See Arts. **276** and **277.**

(20) See rule **1**, Art. **17.** Coefficient of expansion = .0000823. Increase in temperature = $307° - 60° = 247°$. Consequently the expansion is $250 \times 247 \times .0000823 = 5.082$ in. Ans.

(21) See Art. **67.** Volume required is 18 cubic feet for each thousand pounds; therefore, for 25 tons the volume is

$$25 \times 18 \times 2 = 900 \text{ cu. ft.}$$

Length of bin is 18 feet, depth 6 feet; consequently, the width is

$$\frac{900}{18 \times 6} = 8\tfrac{1}{3} \text{ ft.} = 8 \text{ ft. } 4 \text{ in.} \quad \text{Ans.}$$

(22) See Table 15, Art. **96.** Each square foot of an 8-inch brick wall conducts $.46 \times 70 = 32.2$ B. T. U. from the air in the room to the outer atmosphere in one hour. Consequently, the total loss from the brick wall will be

$$875 \times 32.2 = 28{,}175 \text{ B. T. U. per hour.}$$

Excess of loss per hour equals
28,175 B. T. U. − 6,125 B. T. U. = 22,050 B. T. U. Ans.

(23) See Art. **119.**

(24) See Art. **142.**

(25) See Arts. **172** to **176.**

(26) See Art. **199.**

(27) (*a*) When water is run into the system, air will accumulate in return pipes under the left-hand bench; consequently, the water will not circulate through the greenhouse. (*b*) To remedy the defect, an air-vent pipe should be taken from the highest point of the air lock and led up into the main *a*, or over and into the top of the expansion tank.

(28) See Art. **244.**

(*a*) Loss of cooling is 136,000 B. T. U. per hour. Total loss by cooling and ventilation equals

$$136{,}000 \times 2.18 = 296{,}480 \text{ B. T. U. per hour.}$$

The efficiency of the furnace being 50%, it follows that 50% of 13,400, or 6,700 B. T. U., are utilized for each pound of coal burned; therefore, amount of coal required to heat the building for a period of 24 hours equals

$$24 \times \tfrac{296480}{6700} = 1{,}056 \text{ lb., nearly, or about}$$
$$44 \text{ lb. per hour.} \quad \text{Ans.}$$

(*b*) Size of grate equals

$$\tfrac{44}{4} = 11 \text{ sq. ft.} \quad \text{Ans.}$$

(*c*) Area of heating surface is 45 to 1 of grate surface; consequently, area of heating surface =

$$11 \text{ sq. ft.} \times 45 = 495 \text{ sq. ft.} \quad \text{Ans.}$$

(29) See Art. **279.** Mixing valves should be used.

(30) See rule **2**, Art. **21.**
$U = 20 \times .23751 \times 160 = 760.032$ B. T. U. Ans.

(31) See Arts. **71** and **73.**

(32) See rule **9**, Art. **101.**

$$\text{Temperature} = \frac{275{,}000 \times 58}{319{,}000} + 70° = 120°. \quad \text{Ans.}$$

(33) (*a*) See Art. **122.**
(*b*) See Art. **126** and Fig. 49.
(*c*) See Art. **123** and Fig. 46.

(34) See Arts. **149** and **150.**

(35) See Art. **177.**

(36) See Arts. **201** and **202.**

(37) (*a*) See Art. **225.**
(*b*) See Art. **221.**

(38) See Arts. **247** to **250.**

(39) See Art. **298.**

(40) See rule **3**, Art. **29.** From Table 7 the heat contained in 1 cu. ft. at 140° is 2.2013 B. T. U.

Applying the rule, heat given off equals

$$\frac{300 \times (140 - 60) \times 2.2013}{140} = 377.37 \text{ B. T. U.} \quad \text{Ans.}$$

(41) The continuous flat coil, Art. **75.** This is the most positive circulating and most simple in construction that can be devised for the conditions specified.

(42) See Art. **103.**

(43) See Art. **125.**

(44) See Arts. **153** and **154.**

(45) See Art. **178.**

(46) See Arts. **205** and **206.**

(47) See Art. **224.**

(48) See Art. **256.** Two parts of CO_2 per 10,000 parts of air in excess of that contained by the outer atmosphere.

(49) See Arts. **299** to **301.**

(50) Using rule 4,

$$V_1 = \frac{100(460+160)}{460+0} = 134.8 \text{ cu. ft.} \quad \text{Ans.}$$

(51) (*a*) See Art. **78.**

(*b*) See Art. **79.**

(52) See Arts. **105** to **108.**

(53) See Art. **128.**

(54) (*a*) See Arts. **142** and **156.**

(*b*) See Art. **143.**

(55) See Art. **181.**

(56) See Art. **207.**

(57) See Art. **227.** The ratio between grate area and furnace-heating surface should average 1 to 50; consequently, grate area = 6.5 sq. ft. × 50 = 325 sq. ft. Ans.

(58) See Art. **260.**

(59) See Art. **303.**

(60) (*a*) See Arts. **43** to **47.**

(*b*) See Arts. **48** to **50.**

(*c*) See Arts. **51** and **52.**

(61) See Art. **83.**

(62) See end of Art. **109.**

(63) See Art. **129.** Since the boiler must make steam readily, without a rapid rate of combustion, we should use the largest numbers given in the table. Consequently,

(*a*) Heating surface = 18 × 15 = 270 sq. ft. Ans.

(*b*) Heating surface = 12 × 15 = 180 sq. ft. Ans.

(*c*) Heating surface = 14 × 15 = 210 sq. ft. Ans.

(64) See Arts. **157** and **158.**

(65) See Arts. **182, 187,** and **188.**

(66) See Art. **208.**

(67) See Art. **228.**

(68) See Arts. **262** and **263.**

(69) See Arts. **277** and **304.**

(70) (*a*) See Arts. **58** to **61.**

(*b*) See Art. **60.**

(71) See Art. **84.**

(72) See Art. **112.**

(73) See Art. **131.**

(74) See rule **10**, Art. **162.**

Diameter $= \sqrt{\frac{4900}{100} \div .7854} = 8$ in., nearly. Ans.

(75) See Arts. **189** and **190.** If they are not connected at the top, the sections will become air-bound, and there will be no circulation through them.

(76) See Art. **211.** According to the table,

heating surface $= 84 \times 1.70 = 142.8$ sq. ft. Ans.

(77) See Arts. **230** and **245.**

(78) See Arts. **264** and **265.**

(79) See Art. **305.**

(80) See rule **5**, Art. **39.** Weight of 1 cubic foot of steam at 35° (see Table 8) is .000325. Weight of 1 cubic foot at 75° is .00135. Hence, relative humidity equals

$.000325 \div .00135 = .2407$, or 24.07 per cent. Ans.

(81) See Table 11, Art. **85.** (*a*) The room *A* is to be heated with a radiator having single tubes 40 inches high. According to the table, each square foot will emit

$$1.85(220 - 70) = 277.5 \text{ B. T. U. per hour.}$$

The surface of the radiator should be

$$\frac{30{,}525}{277.5} = 110 \text{ sq. ft.} \quad \text{Ans.}$$

(*b*) In like manner, each square foot of surface in the low radiator in room *B* will emit

$$2.53(220 - 70) = 379.5 \text{ B. T. U. per hour,}$$

and the surface of this radiator should be

$$\frac{30,525}{379.5} = 80.5 \text{ sq. ft., nearly.} \quad \text{Ans.}$$

(82) See Art. **113.**

(83) See Art. **132.**

(84) See Art. **166.**

(85) No. See Art. **192.**

(86) See Table 18, Art. **215.** With a circuit of 300 feet, a 4-inch main will supply 390 square feet direct radiation on first floor, or (see table of factors, Art. **211**) $390 \times 1.70 = 693$ square feet on third floor; or it will supply 100 square feet on first floor, and $290 \times 1.70 = 493$ square feet on third floor. The extra surface which the pipe will supply on the third floor, then, will be about $493 - 300 = 193$ sq. ft. Ans.

(87) See Arts. **231** and **232.**

(88) See Arts. **266** to **268.**

(89) See Arts. **284** and **285.**

(90) See Art. **42.**

(91) See Table 14, Art. **89.** With a velocity of 20 feet per second, the air will receive from the heater 9 B. T. U. per square foot of heating surface per hour per degree difference. Consequently, each square foot of surface will emit

$$9(210 - 60) = 1,350 \text{ B. T. U. per hour.}$$

Since there are 3,000 square feet in the heater, the total emission per hour equals

$$3,000 \times 1,350 = 4,050,000 \text{ B. T. U.} \quad \text{Ans.}$$

(92) See Art. **115.**

(93) See Arts. **135** and **136.**

(94) See Art. **167.**

(95) See Art. **193.**

(96) By reference to Tables 18 and 19, the pipes will be proportioned in Fig. 1.

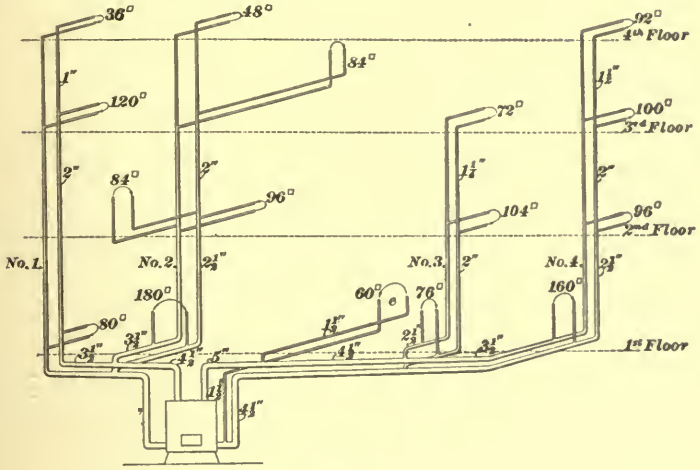

FIG. 1.

(97) See Art. **233.**

(98) See Art. **269.**

(99) See Art. **280.**

(100) See Arts. **63** and **64.**

PAINTING AND DECORATING.

(QUESTIONS 1–75. SEC. 18.)

---

(1) See Art. **88.**

(2) (*a*) See Art. **54.**
(*b*) See Art. **56.**

(3) See Art. **177.**

(4) See Arts. **92** and **93.**

(5) (*a*) See Art. **40.**
(*b*) See Arts. **4** and **37.**

(6) See Art. **177.**

(7) Pound brushes. See Art. **98.**

(8) See Art. **6.**

(9) See Art. **183.**

(10) See Art. **99.**

(11) See Art. **12.**

(12) See Art. **13.**

(13) See Art. **100.**

(14) See Arts. **101** and **103.**

(15) (*a*) See Art. **16.**
(*b*) See Art. **15.**

(16) See Art. **62.**

(17) See Art. **103.**

(18) See Art. **17.**

(19) See Art. **169.**

(20) See Art. **106.**

(21) See Art. **22.**

(22) See Art. **114.**

(23) (*a*) See Art. **107.**
(*b*) See Art. **106.**

(24) See Art. **20.**

(25) See Art. **105.**

(26) See Art. **114.**

(27) See Art. **101.**

(28) See Arts. **119** to **129.**

(29) (*a*) See Art. **27.**
(*b*) See Art. **29.**

(30) See Art. **60.**

(31) See Arts. **150** to **152.**

(32) See Art. **69.**

(33) See Art. **166.**

(34) See Art. **39.**

(35) See Art. **143.**

(36) See Art. **86.**

(37) See Art. **43.**

(38) See Arts. **155** and **159.**

(39) See Art. **170.**

(40) See Art. **32.**

(41) See Art. **119.**

(42) See Art. **81.**

(43) See Art. **74.**

(44) (*a*) See Art. **174.**
(*b*) See Art. **175.**

(45) See Art. **124.**

(46) See Art. **142.**

(47) See Art. **33.**

(48) See Art. **52.**

(49) See Art. **142.**

(50) See Art. **70.**

(51) See Art. **70.**

(52) See Art. **166.**

(53) See Art. **69.**

(54) See Art. **19.**

(55) See Art. **147.**

(56) See Art. **167.**

(57) See Art. **38.**

(58) See Arts. **47** to **51.**

(59) See Art. **53.**

(60) See Art. **132.**

(61) See Art. **148.**

(62) (*a*) See Art. **65.**
(*b*) See Art. **67.**

(63) See Art. **154.**

(64) See Art. **61.**

(65) See Art. **166.**

(66) See Art. **62.**

(67) See Art. **161.**

(68) See Art. **43.**

(69) See Art. **69.**

(70) See Art. **169.**

(71) See Art. **171.**

(72) See Art. **117.**

(73) See Art. **68.**

(74) See Art. **59.**

(75) See Art. **87.**

Estimating and Calculating Quantities.

(QUESTIONS 1–56. SEC. 19.)

(1) (*a*) and (*b*) See Art. **24.** Table.

(2) (*a*) and (*b*) See Art. **28.**

(3) (*a*) 62 cts. See Art. **11.**
(*b*) 12 cts. in addition to hauling. See Art. **11.**

(4) (*a*) and (*b*) See Art. **59.**

(5) (*a*) and (*b*) See Arts. **15** and **16.**

(6) See Art. **53.**

(7) See Art. **65** (cost of 1 sq. yd. of cellar floor).

(8) See Art. **49.**

(9) See Art. **68.**

(10) (*a*) and (*b*) See Art. **56.**

(11) See "Cost," Art. **67.**

(12) See Art. **10,** also Art. **65.**

(13) See Art. **25.**

(14) (*a*) and (*b*) See Art. **26.**

(15) See Art. **22.**

(16) (*a*) and (*b*) See Art. **7.**

(17) (*a*) 575. See Art. **13.**
(*b*) 39. See Art. **13.**

(18) See Art. **69.**

(19) (*a*) Granite, 6 sq. ft.; bluestone, 8 sq. ft.; sandstone, 10 sq. ft.
(*b*) 25%.

(20) (*a*) See Art. **21.**
(*b*) See Art. **22.**

(21) (*a*) and (*b*) See Art. **2.**

(22) (*a*) and (*b*) See Art. **46.**

(23) (*a*) About 1½%. See Art. **48.**
(*b*) See Art. **48.**

(24) (*a*) and (*b*) See Art. **26.**

(25) (*a*) and (*b*) See Art. **60.**

(26) See Arts. **44** and **45.**

(27) (*a*) and (*b*) See Art. **4.**

(28) (*a*) and (*b*) See Art. **41.**

(29) (*a*) and (*b*) See Art. **5.**

(30) (*a*) and (*b*) See Art. **27.**

(31) (*a*) and (*b*) See Art. **17.**

(32) (*a*), (*b*), and (*c*) See Art. **62.**

(33) (*a*) and (*b*) See Arts. **42** and **43.**

(34) (*a*) and (*b*) See Art. **61.**

(35) (*a*) and (*b*) See Art. **26.**

(36) (*a*) and (*b*) See Art. **40.**

(37) See Art. **65.**

(38) (*a*) and (*b*) See Art. **51.**

(39) (*a*), (*b*), and (*c*) See Art. **23.**

(40) (*a*) and (*b*) See Arts. **17** and **18.**

(41) (*a*) and (*b*) See Art. **59.**

(42) (*a*) and (*b*) See Art. **38.**

(43) (*a*) and (*b*) See Art. **9.**

(44) (*a*), (*b*), and (*c*) See Art. **20.**

(45) (*a*) and (*b*) See Art. **47.**

(46) (*a*), (*b*), and (*c*) See Art. **14.**

(47) See Art. **41.**

(48) (*a*), (*b*), and (*c*) See Art. **6.**

(49) See Art. **27.**

(50) (*a*), (*b*), and (*c*) See Art. **19.**

(51) (*a*), (*b*), and (*c*) See Arts. **36** and **37.**

(52) (*a*) and (*b*) See Art. **13.**

(53) (*a*) and (*b*) See Arts. **31** and **34.**

(54) See Art. **39.**

(55) See Art. **40.**

(56) (*a*) and (*b*) See Art. **58.**

HISTORY OF ARCHITECTURE.

(QUESTIONS 1–89. SEC. 20.)

---

(1) See Art. **91.**

(2) See Art. **25.**

(3) See Art. **110.**

(4) See Art. **1.**

(5) See Art. **30.**

(6) See Art. **7.**

(7) See Art. **1.**

(8) See Art. **171.**

(9) See Art. **32.**

(10) See Art. **20.**

(11) See Art. **51.**

(12) See Art. **165.**

(13) See Art. **1.**

(14) See Art. **178.**

(15) See Art. **123.**

(16) See Art. **21.**

(17) See Art. **122.**

(18) See Art. **48.**

(19) See Art. **1.**

(20) See Art. **30.**

(21) See Art. **172.**

(22) See Art. **20.**

(23) See Arts. **142** and **168.**

(24) See Art. **122.**

(25) See Art. **1.**

(26) See Art. **124.**

(27) See Art. **99.**

(28) See Art. **24.**

(29) See Art. **65.**

(30) See Arts. **112** to **118.**

(31) See Art. **72.**

(32) See Art. **112.**

(33) See Art. **33.**

(34) (*a*) See Art. **24.**
(*b*) See Art. **25.**

(35) See Art. **66.**

(36) See Art. **19.**

(37) See Arts. **65** and **66.**

(38) See Arts. **53** and **54.**

(39) See Art. **37.**

(40) See Art. **25.**

(41) See Art. **80.**

(42) See Art. **19.**

(43) See Art. **65.**

(44) See Art. **49.**

(45) See Art. **38.**

(46) See Arts. **90** and **91.**

(47) See Art. **5.**

(48) See Art. **39.**

(49) See Arts. **86** to **88.**

(50) See Arts. **70** and **82.**

(51) See Art. **121.**

(52) See Art. **21.**

(53) See Art. **83.**

(54) See Arts. **18** and **19.**

(55) See Art. **3.**

(56) See Art. **50.**

(57) See Art. **21.**

(58) See Art. **88.**

(59) See Art. **64.**

(60) See Art. **58.**

(61) See Art. **63.**

(62) See Art. **108.**

(63) See Art. **25.**

(64) See Art. **134.**

(65) See Art. **131.**

(66) See Art. **70.**

(67) No.

(68) See Art. **168.**

(69) See Art. **11.**

(70) See Art. **73.**

(71) See Art. **25.**

(72) See Art. **123.**

(73) See Art. **30.**

(74) See Art. **49.**

(75) See Art. **49.**

(76) See Art. **70.**

(77) See Art. **139.**

(78) See Art. **116.**

(79) See Art. **11.**

(80) See Arts. **141** to **151.**

(81) See Art. **26.**

(82) See Art. **111.**

(83) See Art. **43.**

(84) See Art. **27.**

(85) See Arts. **95** and **96.**

(86) See Art. **111.**

(87) See Art. **169.**

(88) See Art. **160.**

(89) See Art. **177.**

ARCHITECTURAL DESIGN.

(QUESTIONS 1–88. SEC. 21.)

(1) See Art. **69.**

(2) See Art. **24.**

(3) See Art. **66.**

(4) See Art. **56.**

(5) See Art. **105.**

(6) See Art. **40.**

(7) See Art. **67.**

(8) See Art. **24.**

(9) See Art. **66.**

(10) See Art. **55.**

(11) See Art. **1.**

(12) See Art. **41.**

(13) See Art. **9.**

(14) See Art. **28.**

(15) See Art. **67.**

(16) See Art. **39.**

(17) See Art. **23.**

(18) See Arts. **41** and **59.**

(19) See Arts. **42** and **43.**

(20) See Art. **71.**

(21) See Art. **84.**

(22) See Art. **128.**

(23) See Art. **1.**

(24) Grouping of parts so as to present an impression of singleness of mass.

(25) See Art. **30.**

(26) See Arts. **45** and **46.**

(27) The Parthenon.

(28) See Arts. **69** to **77.**

(29) See Arts. **109, 113,** and **115.**

(30) See Art. **36.**

(31) See Art. **16.**

(32) See Art. **30.**

(33) See Art. **40.**

(34) See Art. **78.**

(35) See Art. **78.**

(36) See Art. **113.**

(37) See Art. **32.**

(38) See Art. **6.**

(39) See Art. **15.**

(40) See Art. **113.**

(41) See Art. **82.**

(42) See Art. **31.**

(43) See Art. **51.**

(44) See Art. **40.**

(45) See Art. **117.**

(46) See Arts. **86** and **88.**

(47) See Art. **32.**

(48) See Arts. **40, 52, 53,** and **66.**

(49) See Art. **42.**

(50) See Art. **70.**

(51) See Art. **119.**

(52) See Art. **90.**

(53) See Art. **34.**

(54) See Art. **55.**

(55) See Art. **11.**

(56) See Art. **124.**

(57) See Art. **99.**

(58) See Art. **35.**

(59) See Art. **56.**

(60) See Art. **44.**

(61) See Art. **32.**

(62) See Art. **91.**

(63) See Art. **35.**

(64) See Art. **53.**

(65) See Art. **128.**

(66) See Art. **84.**

(67) See Art. **92.**

(68) See Art. **37.**

(69) See Art. **59.**

(70) See Art. **17.**

(71) See Art. **93.**

(72) See Art. **14.**

(73) See Art. **101.**

(74) See Art. **102.**

(75) See Art. **131.**

(76) See Art. **98.**

(77) See Art. **22.**

(78) See Art. **129.**

(79) See Art. **95.**

(80) See Art. **52.**

(81) See Art. **65.**

(82) See Arts. **59** and **60.**

(83) See Art. **64.**

(84) See Art. **97.**

(85) See Art. **63.**

(86) See Art. **87.**

(87) See Art. **2.**

(88) See Art. **39.**

ARCHITECTURAL DESIGN.

(QUESTIONS 1–50. SEC. 22.)

---

(1) See Art. **129.**

(2) See Art. **80.**

(3) See Art. **157.**

(4) See Art. **53.**

(5) See Art. **90.**

(6) See Art. **99.**

(7) See Arts. **10, 13,** and **14.**

(8) See Art. **132.**

(9) See Art. **130.**

(10) See Art. **132.**

(11) See Art. **37.**

(12) See Arts. **69** to **71.**

(13) See Art. **82.**

(14) See Arts. **85** and **88.**

(15) See Art. **15.**

(16) See Art. **36.**

(17) See Art. **67.**

(18) See Art. **48.**

(19) See Art. **63.**

(20) See Art. **61.**

(21) See Art. **92.**

(22) See Art. **95.**

(23) See Art. **84.**

(24) See Art. **136.**

(25) See Art. **146.**

(26) See Art. **99.**

(27) See Art. **85.**

(28) See Art. **66.**

(29) See Art. **100.**

(30) See Art. **130.**

(31) See Art. **130.**

(32) See Art. **156.**

(33) See Art. **19.**

(34) See Arts. **24** and **25.**

(35) See Art. **48.**

(36) See Art. **21.**

(37) See Art. **116.**

(38) See Arts. **134** and **135.**

(39) See Art. **160.**

(40) See Art. **66.**

(41) See Arts. **90** and **91.**

(42) See Art. **24.**

(43) See Art. **26.**

(44) See Art. **11.**

(45) See Art. **68.**

(46) See Art. **68.**

(47) See Art. **70.**

(48) See Art. **78.**

(49) See Art. **83.**

(50) See Art. **84.**

SPECIFICATIONS.

(QUESTIONS 1–48. SEC. 23.)

(1) See Art. **1.**

(2) See Art. **1.**

(3) See Art. **1.**

(4) See Art. **1.**

(5) See Arts. **1** and **3.**

(6) See Art. **12.**

(7) The headings should in general be placed in the order in which the work will be executed. See Art. **1.**

(8) See Art. **1.**

(9) See Art. **2.**

(10) When a separate or special contract is made. See Art. **20.**

(11) See Art. **5.**

(12) See Art. **5.**

(13) See Arts. **12** to **15.**

(14) See Art. **2.**

(15) $\frac{3}{4}$ inch. See Art. **13,** "Ashlar Facing."

(16) See Arts. **20** and **26.**

(17) See Art. **2.**

(18) See Art. **2.**

(19) To be under the frost line. See Art. **12,** "Excavations."

(20) See Art. **14,** "Whitewashing."

(21) See Art. **15,** "Lathing."

(22) See Art. **18,** "Finished Floors."

(23) See Art. **23.**

(24) See Arts. **16** and **18.**

(25) See Art. **12,** "Excavations."

(26) See Art. **25.**

(27) See Art. **14,** "Chimneys."

(28 See Art. **24,** last paragraph.

(29) See Art. **19.**

(30) See Art. **23,** "Joints in Cast-Iron Pipe."

(31) See Art. **27.**

(32) See Art. **28.**

(33) See Art. **28.**

(34) See Arts. **13** and **14.**

(35) See Art. **16,** "Shingles."

(36) See Art. **28.**

(37) See Art. **13,** "Ashlar Facing."

(38) See Art. **18,** "Window Frames."

(39) See Art. **22,** "Leaders," and Art. **23,** "Arrangement of System."

(40) See Art. **20,** "Mantels."

(41) See Art. **23.**

(42) Consult Arts. **12** and **13.**

(43) Consult Art. **19.**

(44) Consult Art. **23.**

(45) Consult Art. **18.**

(46) See Art. **16,** "Shingles."

(47) See Art. **27.**

(48) See Arts. **5** to **11.**

BUILDING SUPERINTENDENCE.

(QUESTIONS 1–53. SEC. 24.)

---

(1) See Art. **1.**

(2) See Art. **23.**

(3) See Art. **14.**

(4) See Arts. **25** and **26.**

(5) See Art. **7.**

(6) See Art. **67.**

(7) See Art. **8.**

(8) See Art. **20.**

(9) See Art. **63.**

(10) See Art. **42.**

(11) See Art. **47.**

(12) See Art. **39.**

(13) See Art. **69.**

(14) See Art. **15.**

(15) See Art. **5.**

(16) See Art. **16.**

(17) See Art. **2.**

(18) See Art. **19.**

(19) See Art. **66.**

(20) See Art. **21.**

(21) See Art. **17.**

(22) See Arts. **28** and **29.**

(23) See Art. **44.**

(24) See Art. **17.**

(25) See Art. **14.**

(26) See Art. **20.**

(27) See Art. **34.**

(28) See Art. **38.**

(29) See Art. **106.**

(30) See Art. **114.**

(31) See Art. **108.**

(32) See Art. **122.**

(33) See Art. **95.**

(34) See Art. **89.**

(35) See Art. **143.**

(36) See Art. **120.**

(37) See Art. **112.**

(38) See Art. **94.**

(39) See Art. **88.**

(40) See Art. **112.**

(41) See Art. **89.**

(42) See Art. **103.**

(43) See Art. **84.**

(44) See Arts. **137** and **138.**

(45) See Art. **93.**

(46) See Art. **128.**

(47) See Art. **140.**

(48) See Art. **96.**

(49) See Art. **87.**

(50) See Art. **99.**

(51) See Art. **118.**

(52) See Art. **142.**

(53) See Art. **144.**

CONTRACTS AND PERMITS.

(QUESTIONS 1–50. SEC. 25.)

(1) See Art. **3.**

(2) See Art. **62.**

(3) See Art. **18.**

(4) See Art. **14.**

(5) See Art. **24.**

(6) See Art. **22.**

(7) See Art. **33.**

(8) See Arts. **4** to **7.**

(9) See Art. **36.**

(10) See Arts. **80** to **84.**

(11) See Art. **58.**

(12) See Art. **64.**

(13) See Art. **14.**

(14) See Art. **60.**

(15) See Arts. **25** and **26.**

(16) See Art. **67.**

(17) See Art. **39.**

(18) See Art. **84.**

(19) See Art. **34.**

(20) See Art. **26.**

(21) See Art. **43.**

(22) See Art. **83.**

(23) See Art. **72.**

(24) See Art. **14.**

(25) See Art. **44.**

(26) See Art. **40.**

(27) See Art. **43.**

(28) See Form I, Art. II.

(29) See Art. **33.**

(30) See Art. **61.**

(31) See Art. **88.**

(32) See Art. **28.**

(33) See Arts. **10, 11,** and **12.**

(34) See Art. **42.**

(35) See Art. **20.**

(36) See Arts. **16** and **17.**

(37) See Art. **7.**

(38) See Art. **54.**

(39) See Art. **26.**

(40) See Art. **71.**

(41) See Art. **49.**

(42) See Art. **13.**

(43) See Art. **2.**

(44) See Art. **63.**

(45) See Arts. **53** and **54.**

(46) See Art. **38.**

(47) See Art. **30.**

(48) See Art. **49.**

(49) See Art. **24.**

(50) See Art. **46.**

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